Specifying Intra-Agent Dynamics: Continuous Evolution

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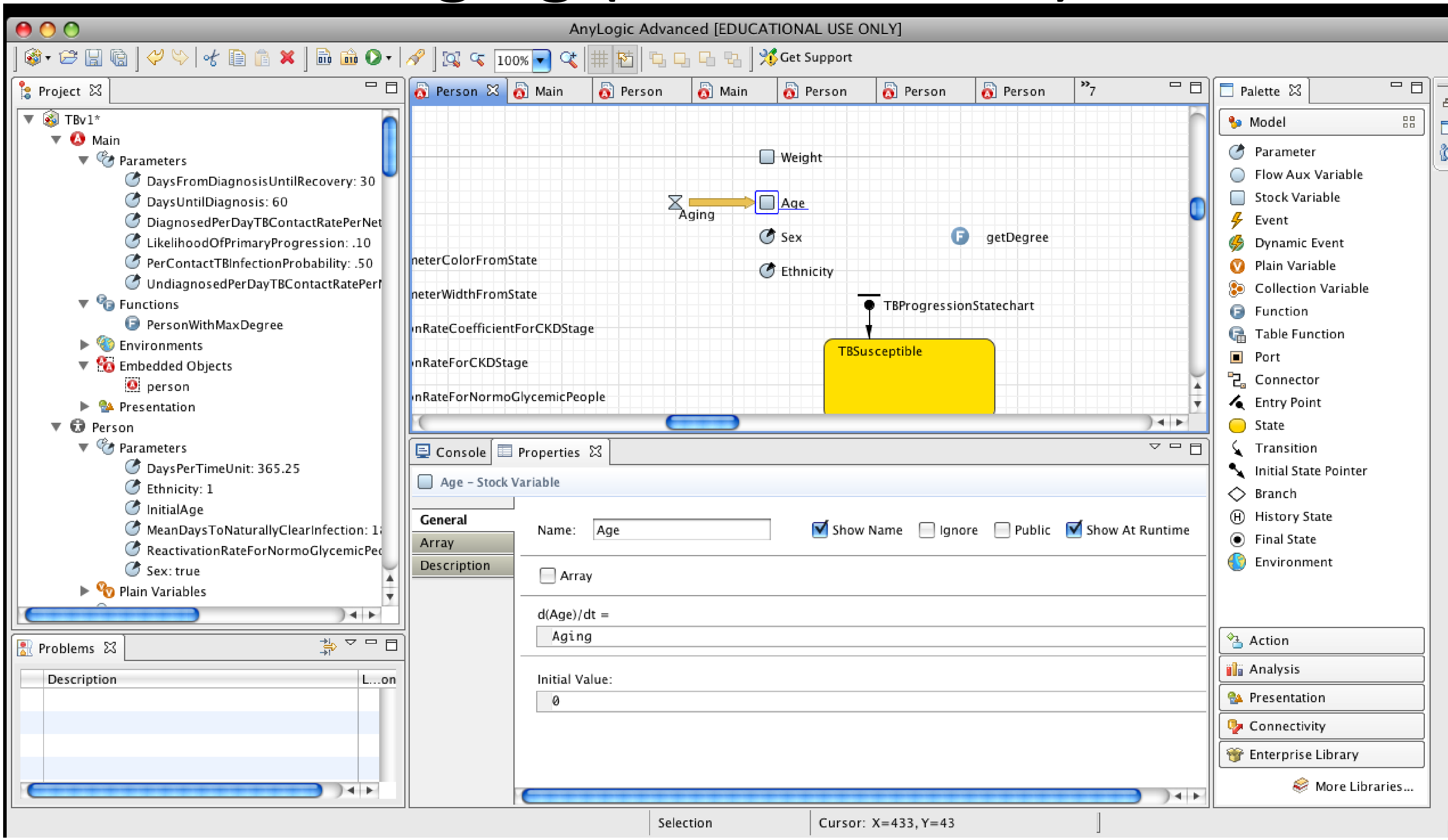
Continuous Agent Behavior

- There are many ways for specifying the behavior of agents
- We have previously focused on situations where
  - The possible states for an agent is some smaller, discrete number (cf. Susceptible, Infected, Recovered)
  - Transitions between states are view as occurring instantaneously (e.g. at the moment of infection)
- Frequently we wish to record continuous changes in state
  - Cf. blood pressure, Age, Weight, pain level, pathogen load, etc.
  - The changes may be fast or slow, but are continuous

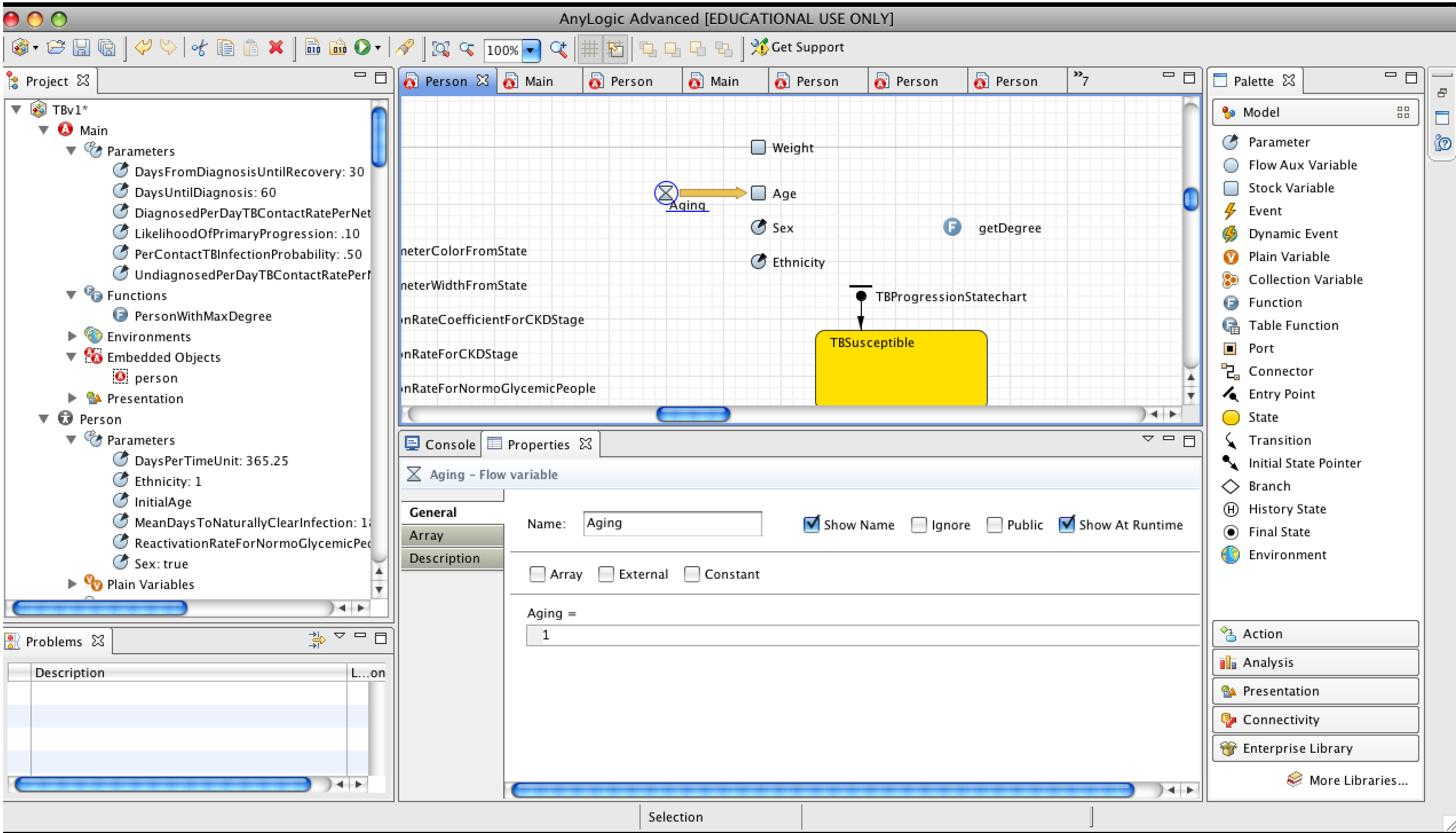
Continuous State Transitions

- We can frequently specify continuous state transitions very conveniently with stocks & flows
- As for state charts, AnyLogic's tools for working with stocks & flow diagrams provide us with a
  - Graphical, high-level picture of key factors in the dynamics
  - A clear indication of what components make up the state
  - A way of editing the detailed assumptions about the transitions

Example of Continuous Dynamics: Aging (Stock Shown)



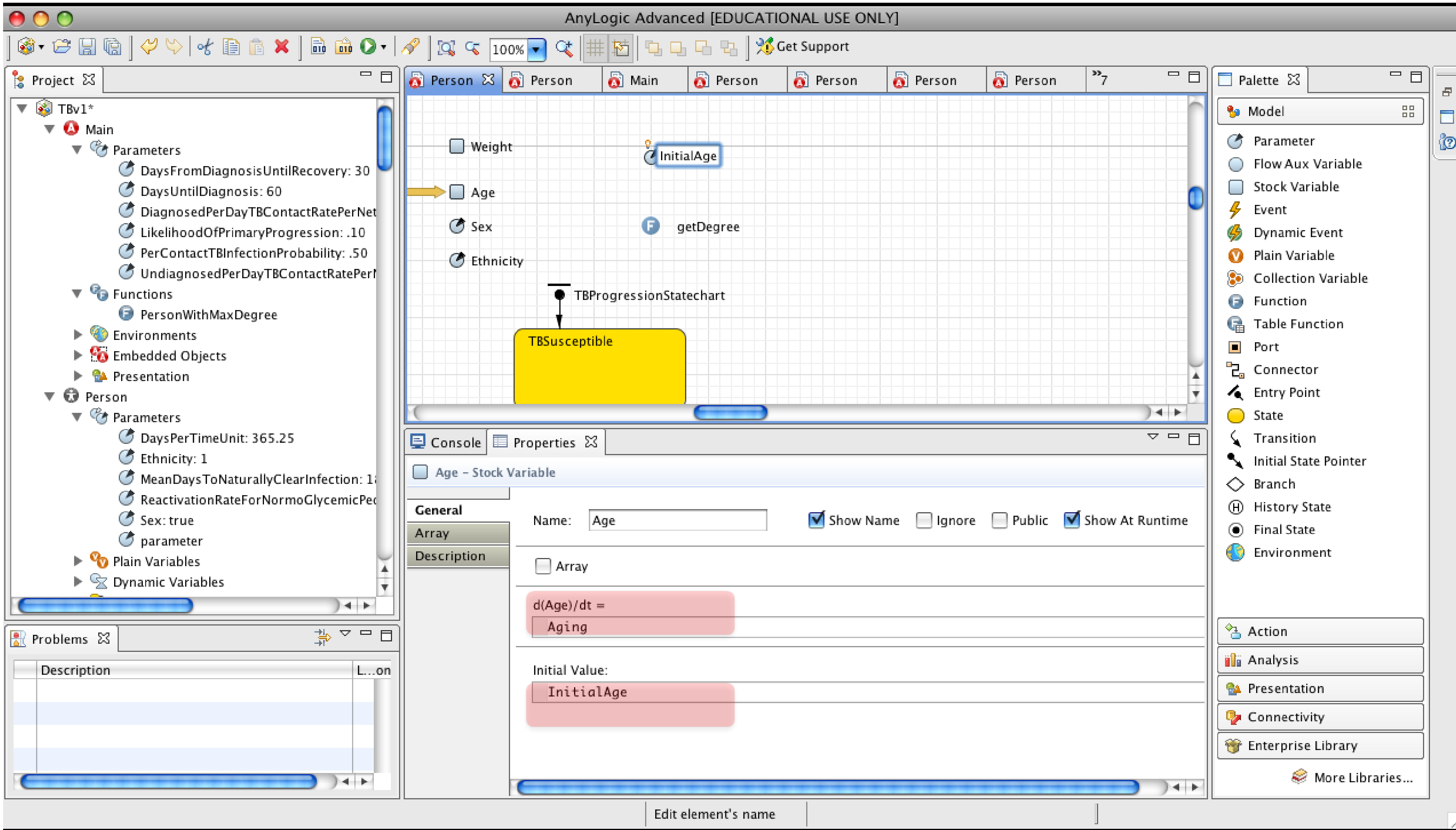
Example of Continuous Dynamics: Aging (Flow Shown)



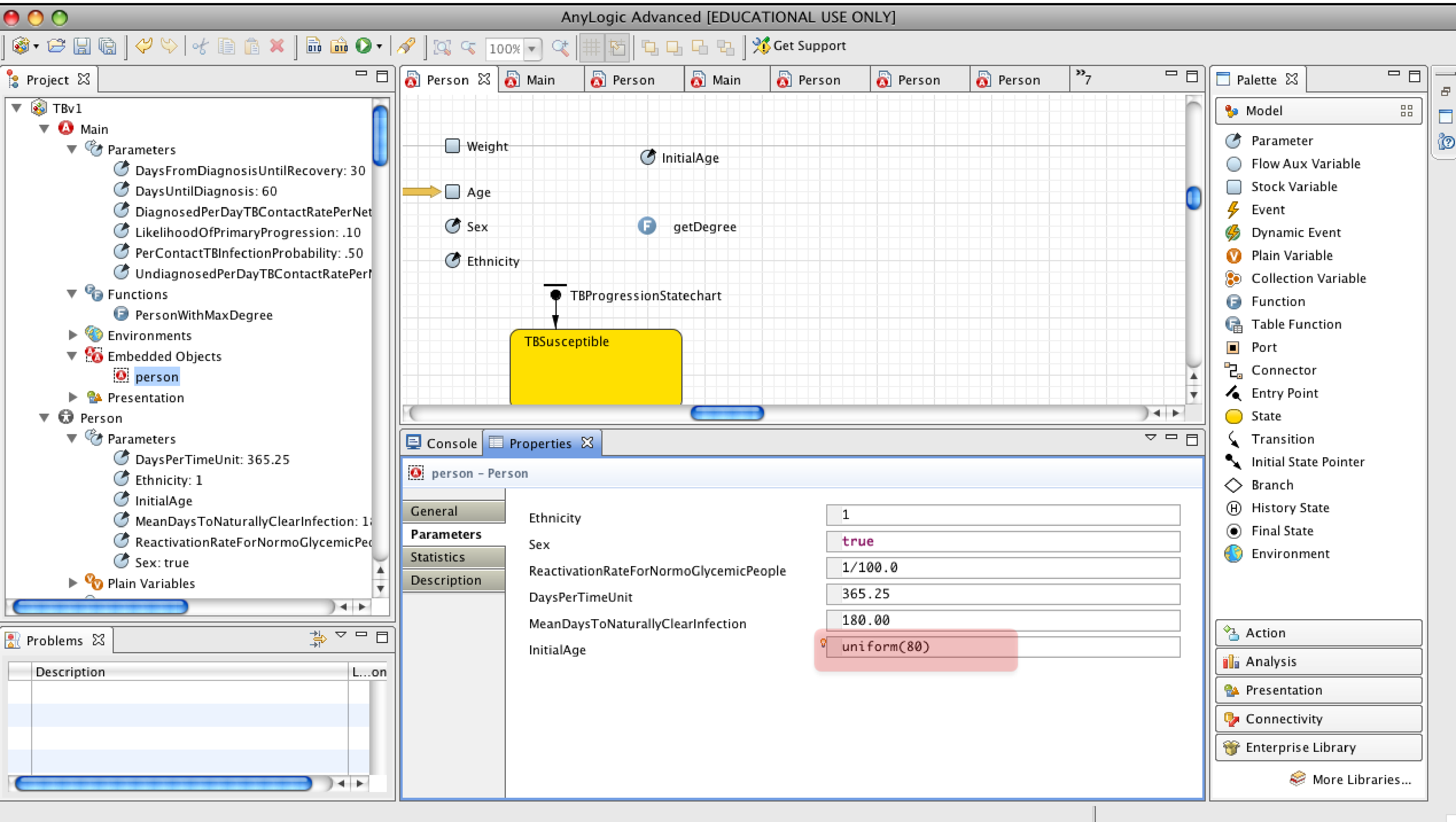
Initialization of a Agent Properties

- We can initialize a population by taking advantage of the properties associated with the Agent population
- This mechanism makes it easy to initialize a heterogeneous population
- The expressions entered in the properties of that population will be evaluated for the corresponding field of each particular agent
  - Common examples might be
    - Using a constant
    - Calling a random number generator Uniform(0,80)
    - Drawing a value from a database

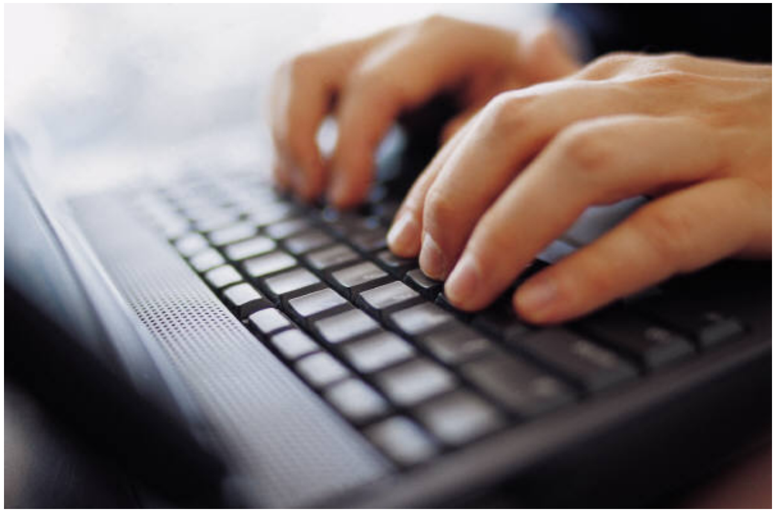
Example: Initialization of Initial Stock Values to Represent an Heterogeneous Population



Making Initial Age Uniformly Distributed between 0 and 80 Years

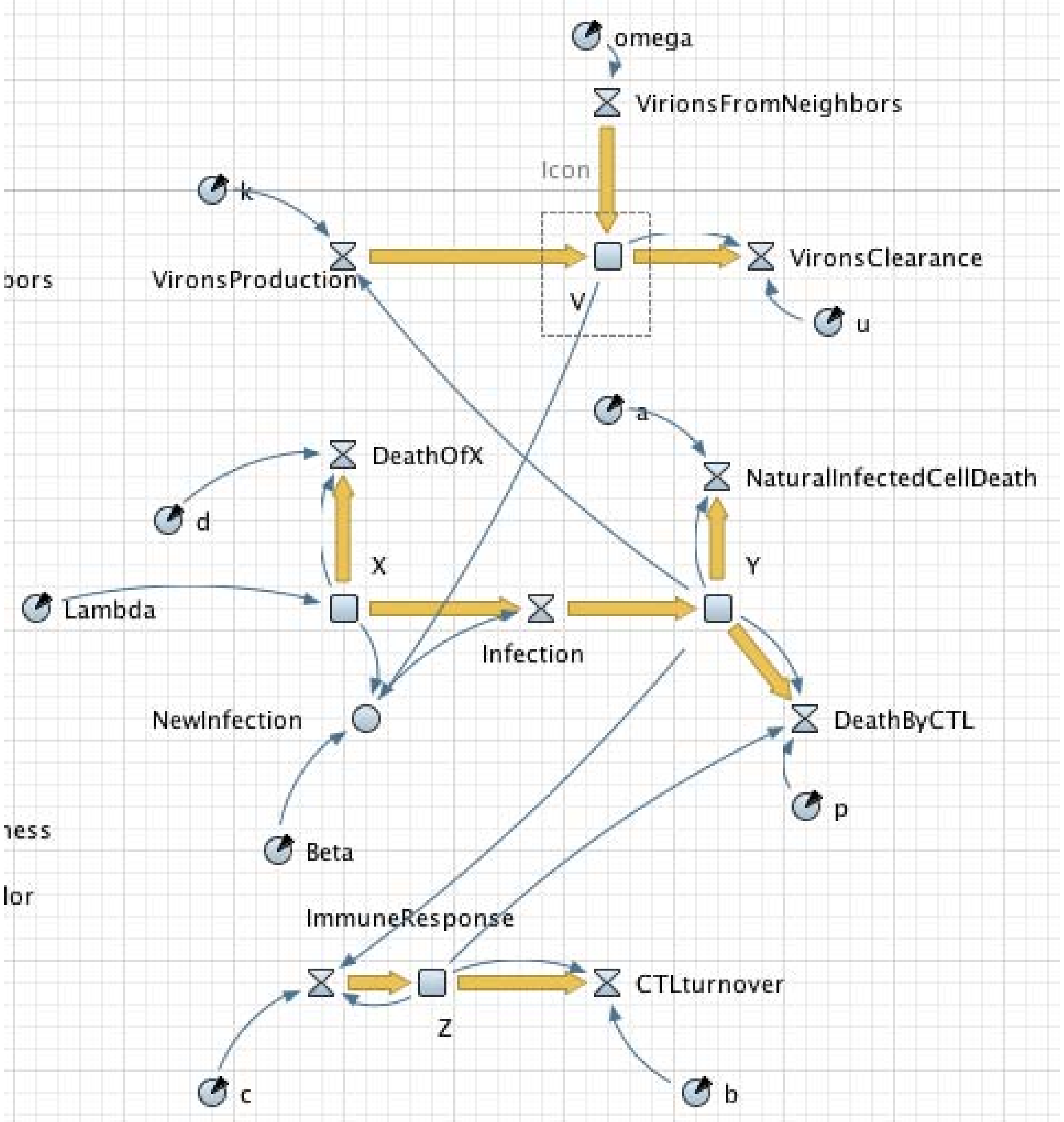


Hands on Model Use Ahead

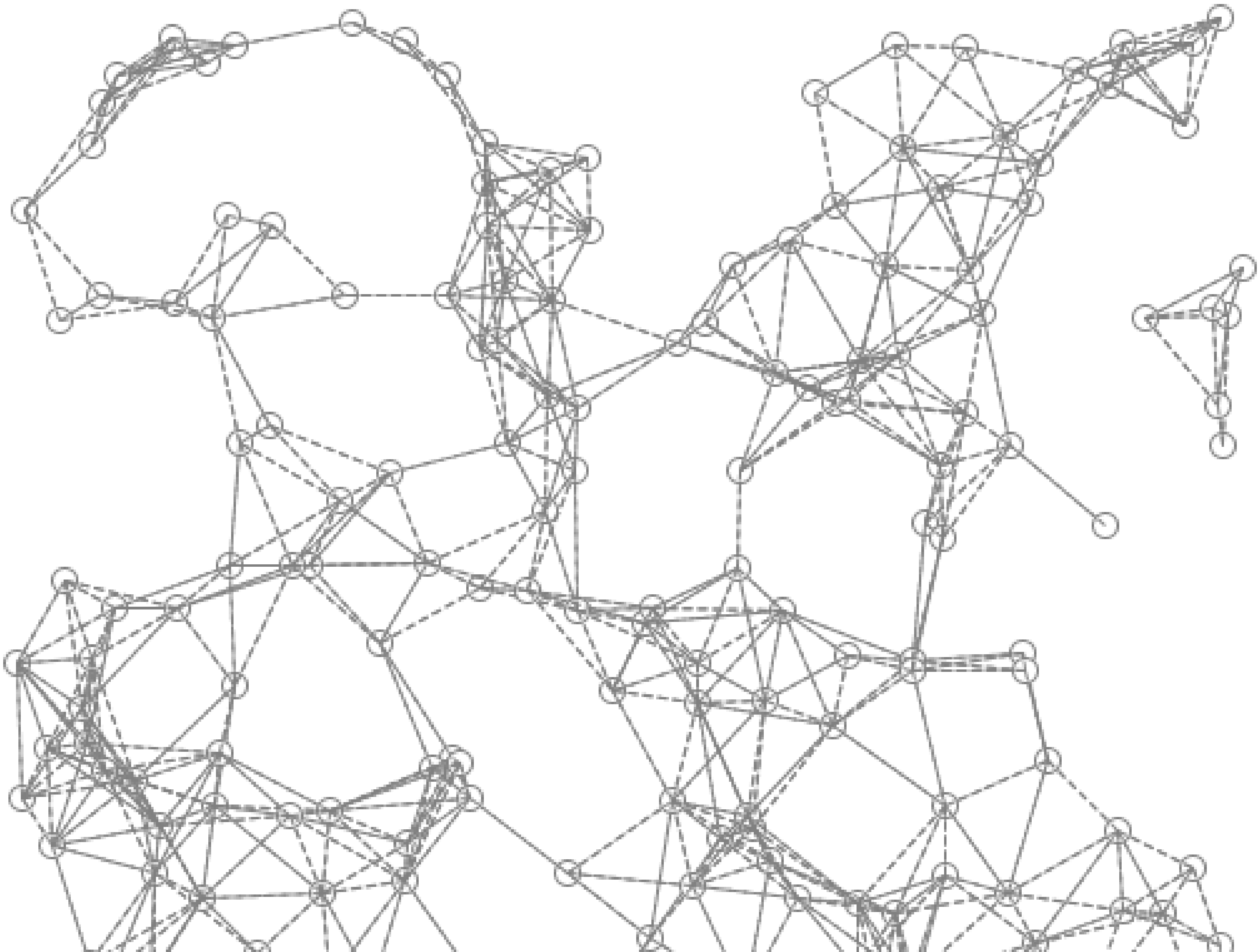


Load model: CTL State Variable V4.alp

Within Host Model of Viral Infections & Resulting Immune Response



Network Embedding of Individuals



State of the System: Stocks (Levels, State Variables, Compartments)

- Stocks (Levels) represent accumulations
  - These capture the "state of the system"
  - Mathematically, we will call these "state variables"
- These can be measured at *one instant in time*
- Stocks are only changed by changes to the flows into & out of them
  - There are no inputs that immediately change stocks
- Stocks are the source of delay in a system

Examples of Stocks

- Water in a tub or reservoir
- People of different types
  - { Susceptible, infective, immune} people
  - Pregnant women
  - Women between the age of x and *y*
  - High-risk individuals
- Healthcare workers
- Medicine in stocks
- Money in bank account
- CO_2 in atmosphere
- Blood sugar
- Stored Energy
- Degree of belief in X
- Stockpiled vaccines
- Goods in a warehouse
- Beds in an emergency room
- Owned vehicles

State Changes: Flows (“Fluxes”, “Rates”)

- If these flow out of or into a stock that keeps track of things of type X, the rates are measured in X/Unit Time (e.g. person/year)
- Typically measure over certain period of time (by considering accumulated quantity over a period of time)
  - e.g. Incidence Rates is calculated by accumulating people over a year, revenue is $/Time, water flow is litres/minute
  - May be *measured* by totalling up over a period of time and dividing by the time
    - Apply to any point in time

Examples of Flows

- Inflow or outflow of a bathtub (litres/minute)
- Rate of incident cases (e.g. people/month)
- Rate of recovery
- Rate of mortality (e.g. people/year)
- Rate of births (e.g. babies/year)
- Rate of treatment (people/day)
- Rate of caloric consumption (kcal/day)
- Rate of pregnancies (pregnancies/month)
- Reactivation Rate (# of TB cases reactivating per unit time)
- Revenue ($/month)
- Spending rate ($/month)
- Power (Watts)
- Rate of energy expenditure
- Vehicle sales
- Vaccine sales
- Shipping rate of goods

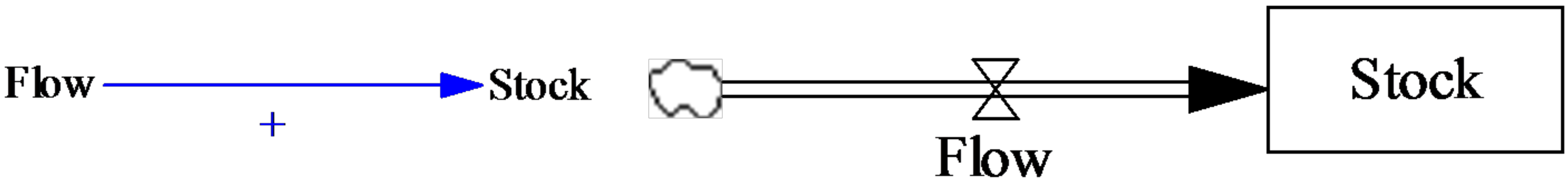
Flows 2

- We can ask conceptually *about* the rate *at* any given point in time – and may change over time
  – Measuring it would have to be over some period
- When speaking about "Rates" for flows, we always mean something measured as X/*Unit Time* (also called a *rate of change over time)*
  – Not all things called "rates" are flows
    - Exchange rate
    - Prevalence rate
    - Rate of return

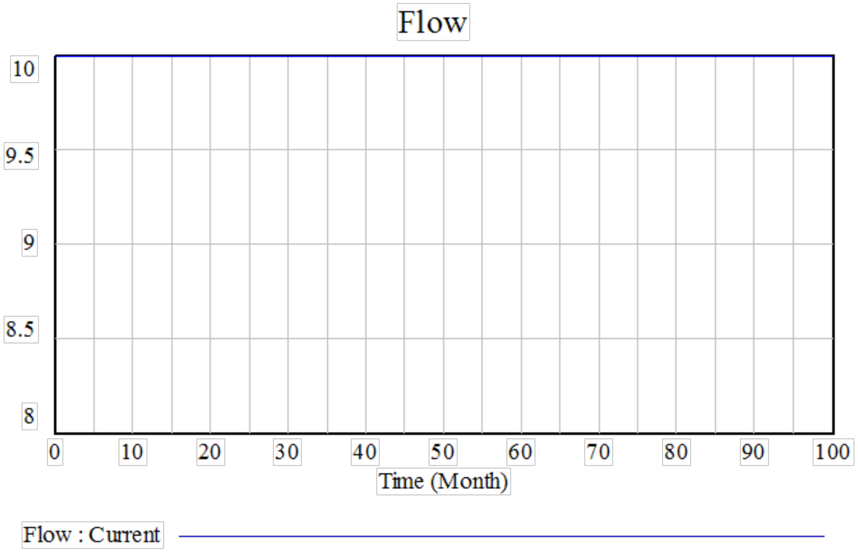
Exercise: Stocks or Flows?

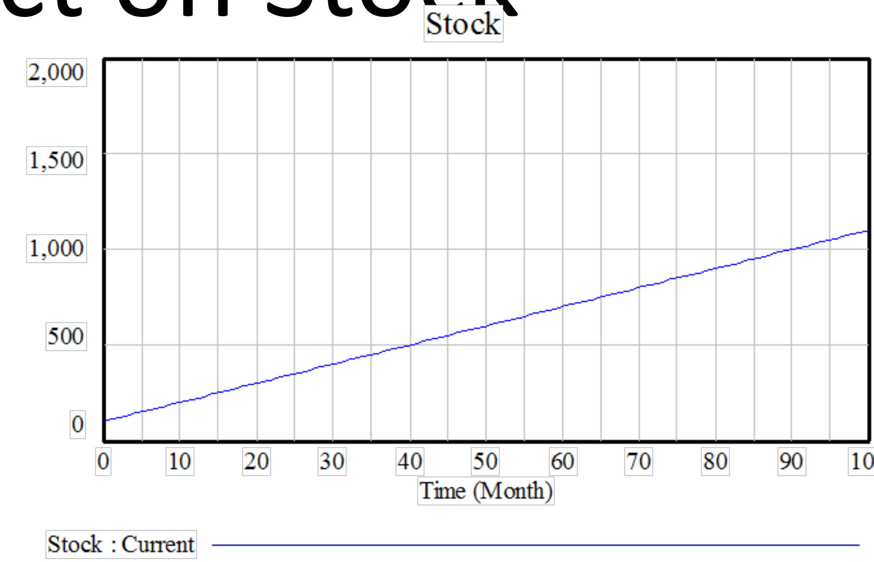
- Account balance
- Income
- Incidence
- Prevalence
- Temperature
- Births
- Profits
- Interest
- Principal
- Shipments
- Car accidents
- Patients on dialysis
- Deaths
- Heart attacks
- Arrests
- Police
- Patients in hospital
- Hospital admissions
- Position
- Speed

Key Component: Stock & Flow

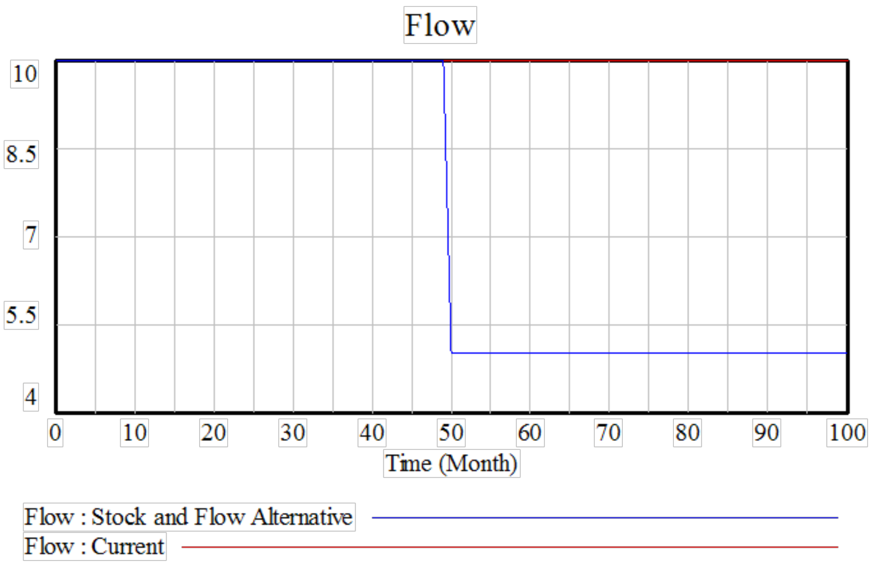


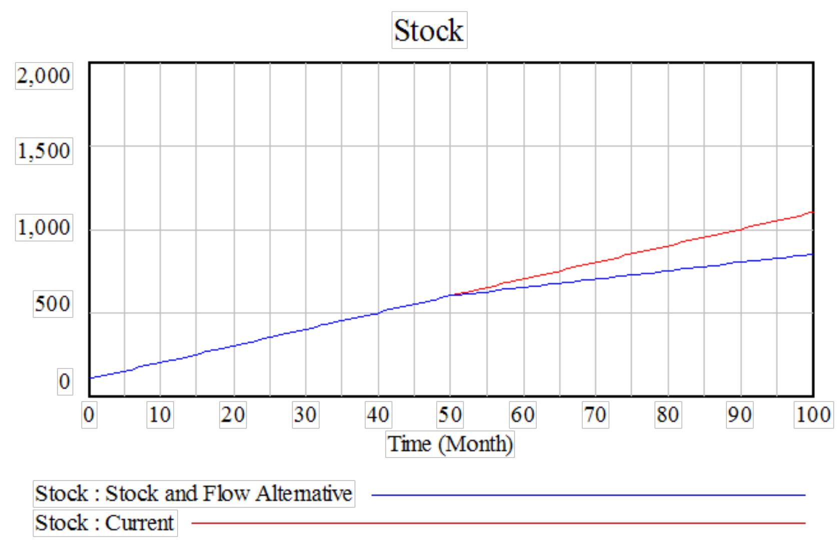
Net Flow Impact on Stock





Impact of Lowering Flow (Rate) to 5/Month?





Loops & Stocks

- Causation does not effect big change instantaneously
  - Loops are not instantaneous
- Stocks only change by changes to the flows into & out of them
  - There are no inputs that immediately change stocks
- All causal loops in the world must involve at least one stock
  - The state of the world must change as part of the process
  - Absent a stock, loop would be instantaneous

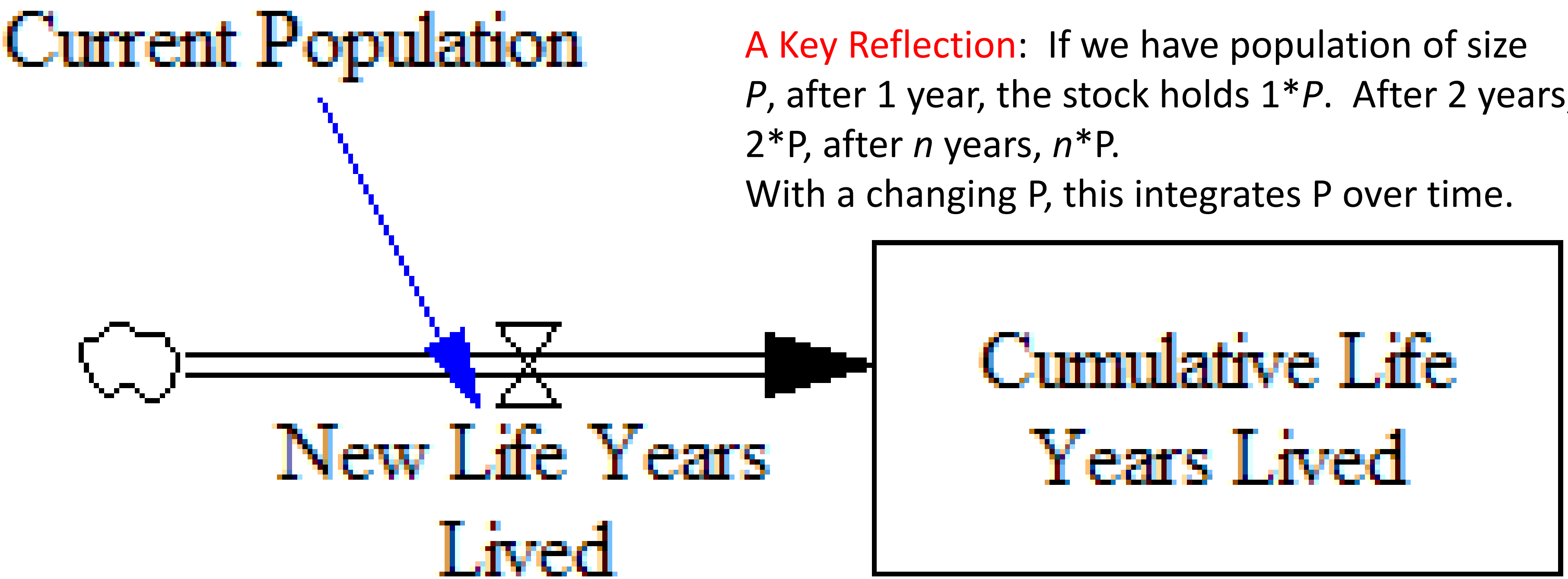
Interactive Steps

- View flows and stocks – does this make sense?
- Hitch up constant “auxiliary” variables to flows
- How does changing constant variables change the stock?

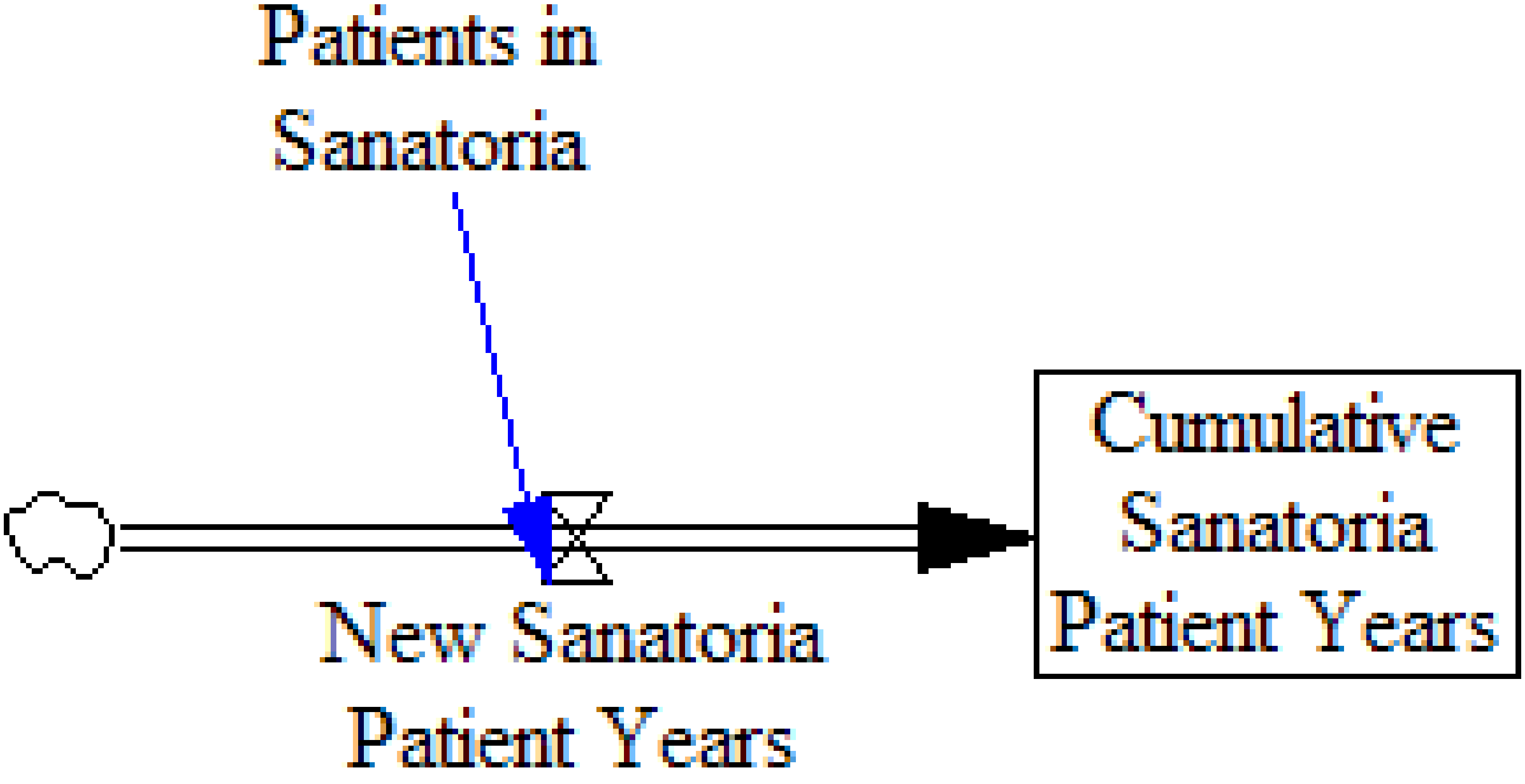
Stocks As Accumulations

- We often use stocks to accumulate (integrate) other (evolving) quantities over time
- Example (assume time measured in years):

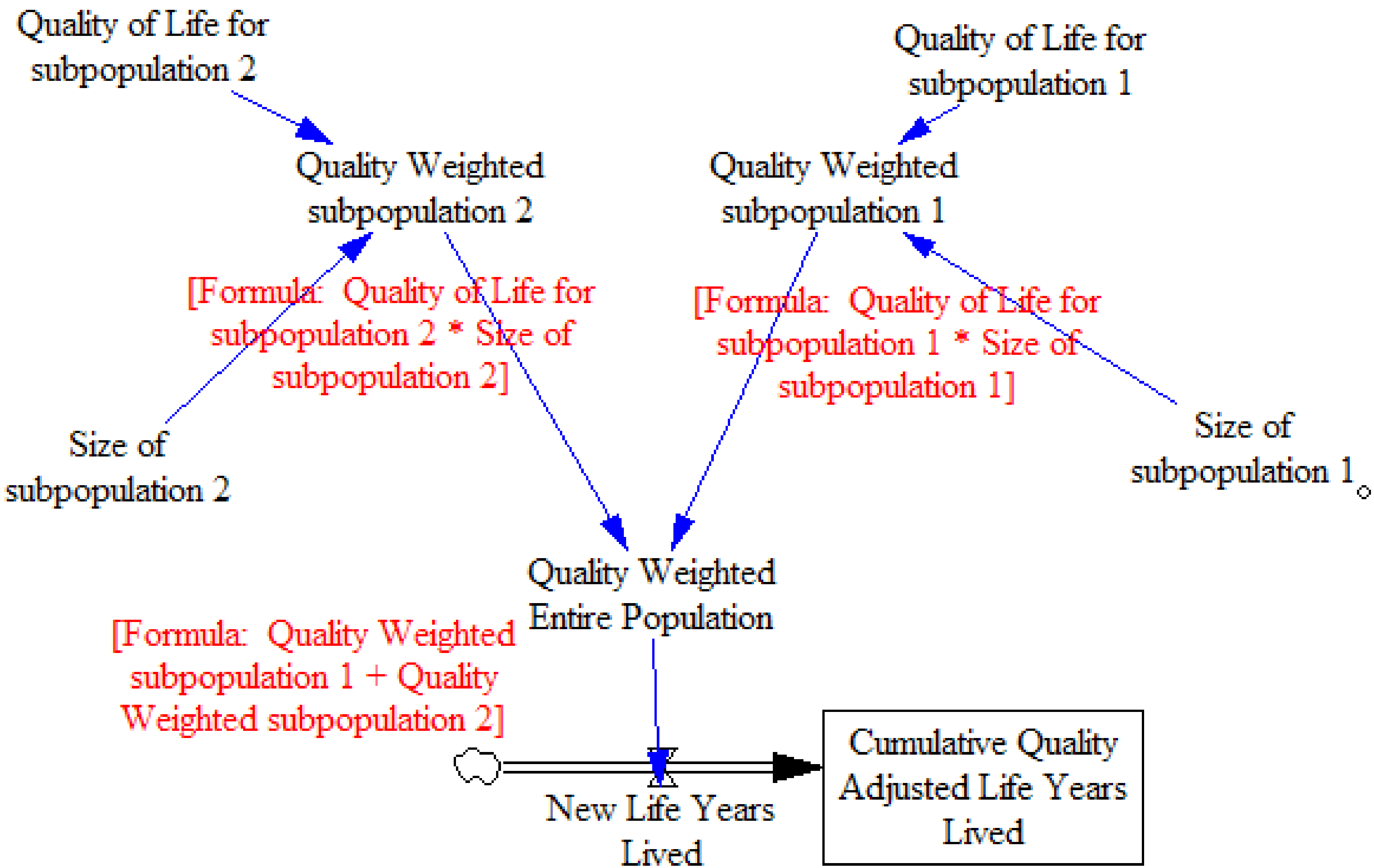
A Key Reflection: If we have population of size *P*, after 1 year, the stock holds 1**P*. After 2 years, 2*P, after *n* years, *n**P.
With a changing P, this integrates P over time.



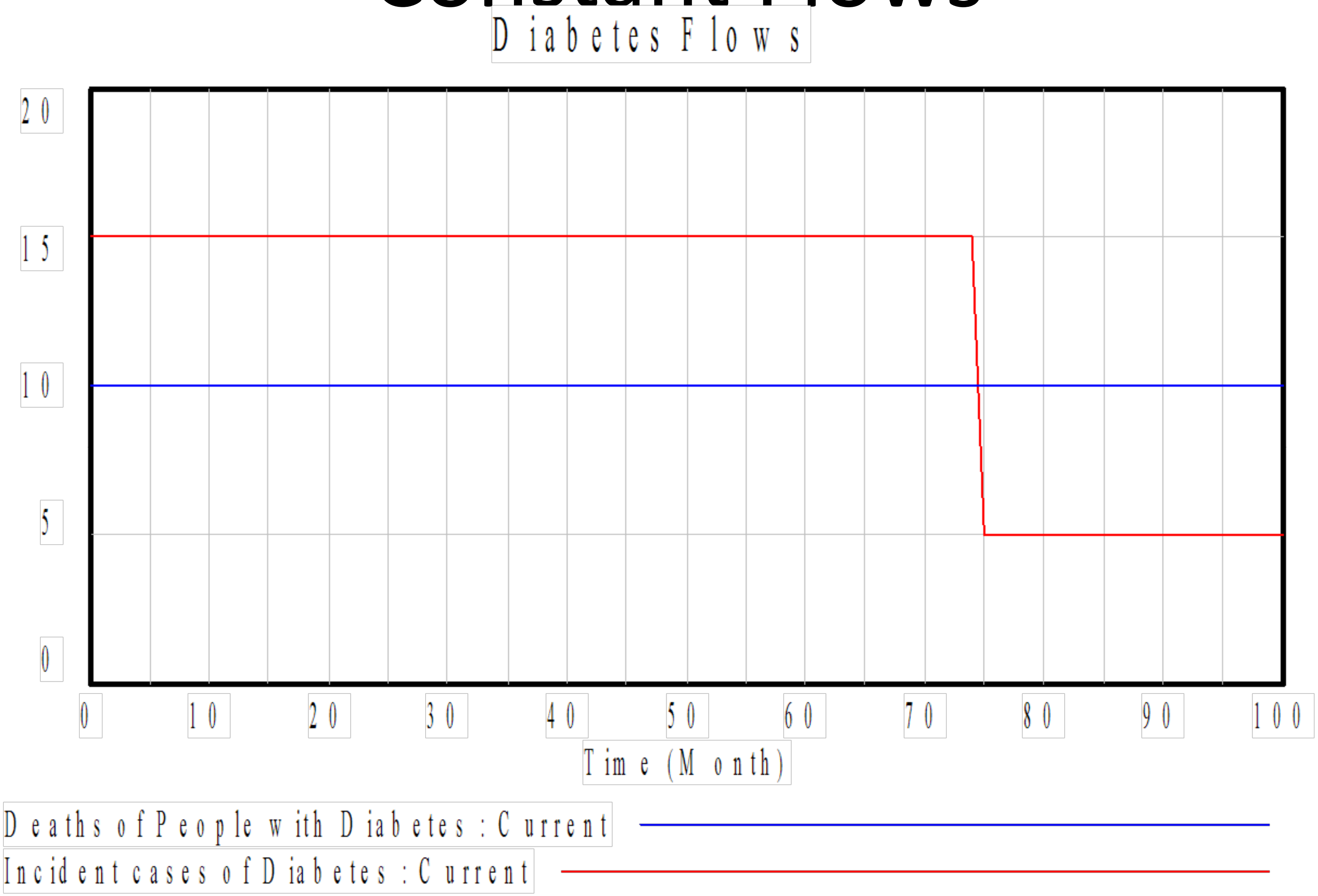
Example 2


Patients in Sanatoria
New Sanatoria Patient Years
Cumulative Sanatoria Patient Years

Slightly more Sophisticated

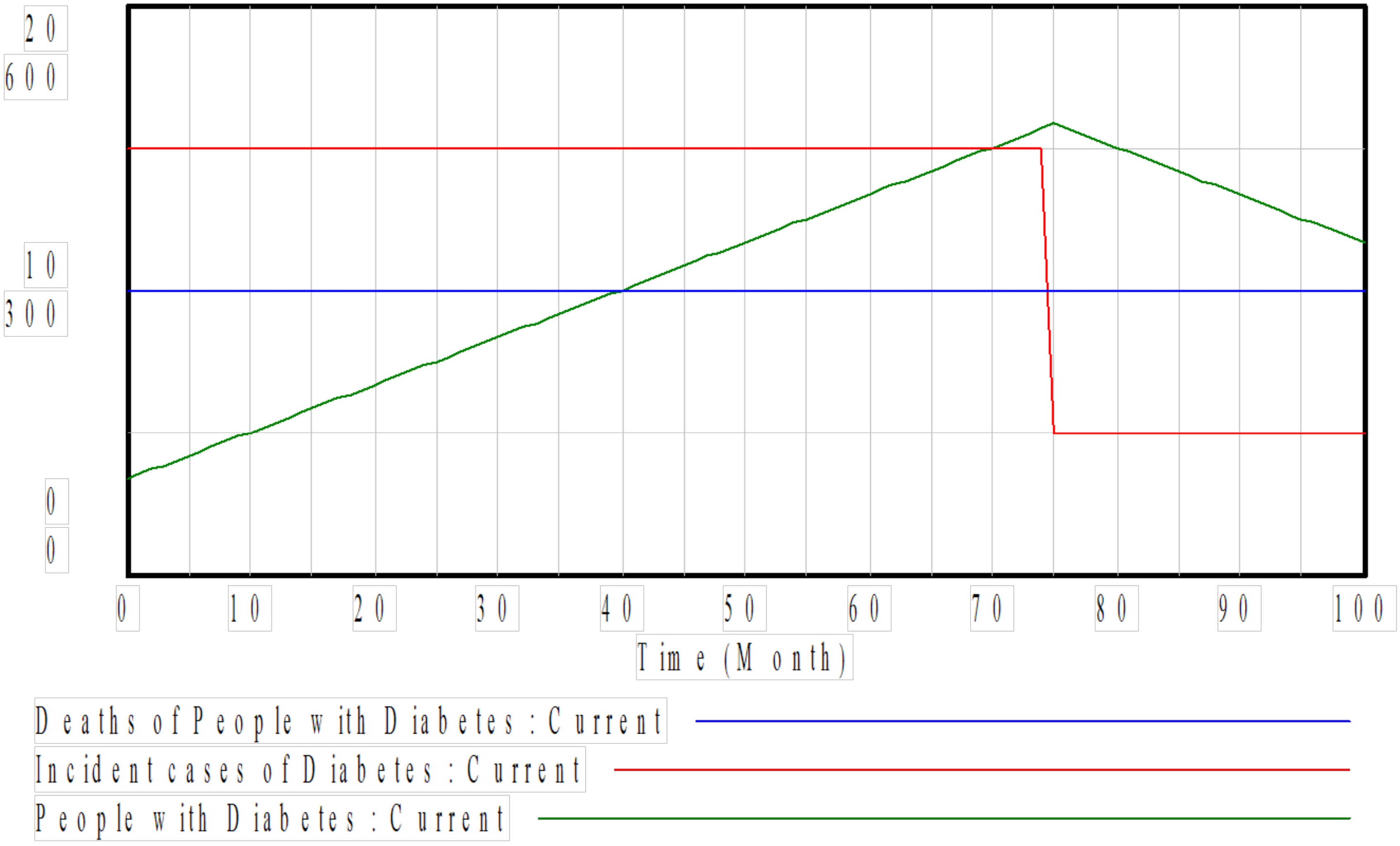


Constant Flows

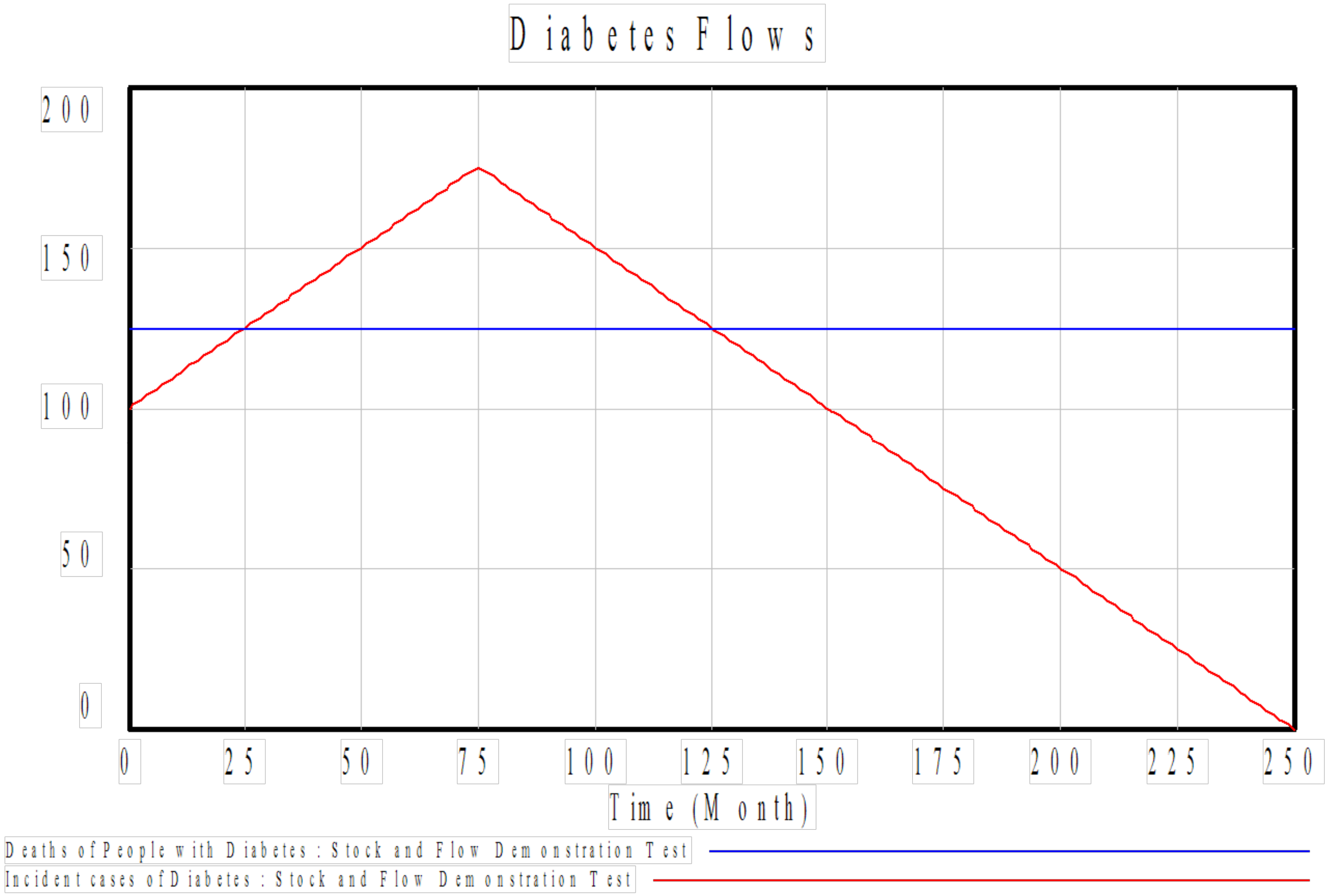


What happens to the stock?

Resulting Stock

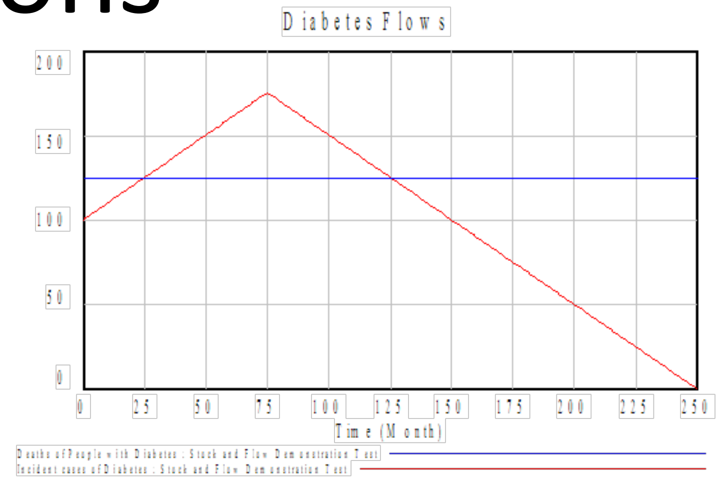


Suppose we have these Flows (Rates)



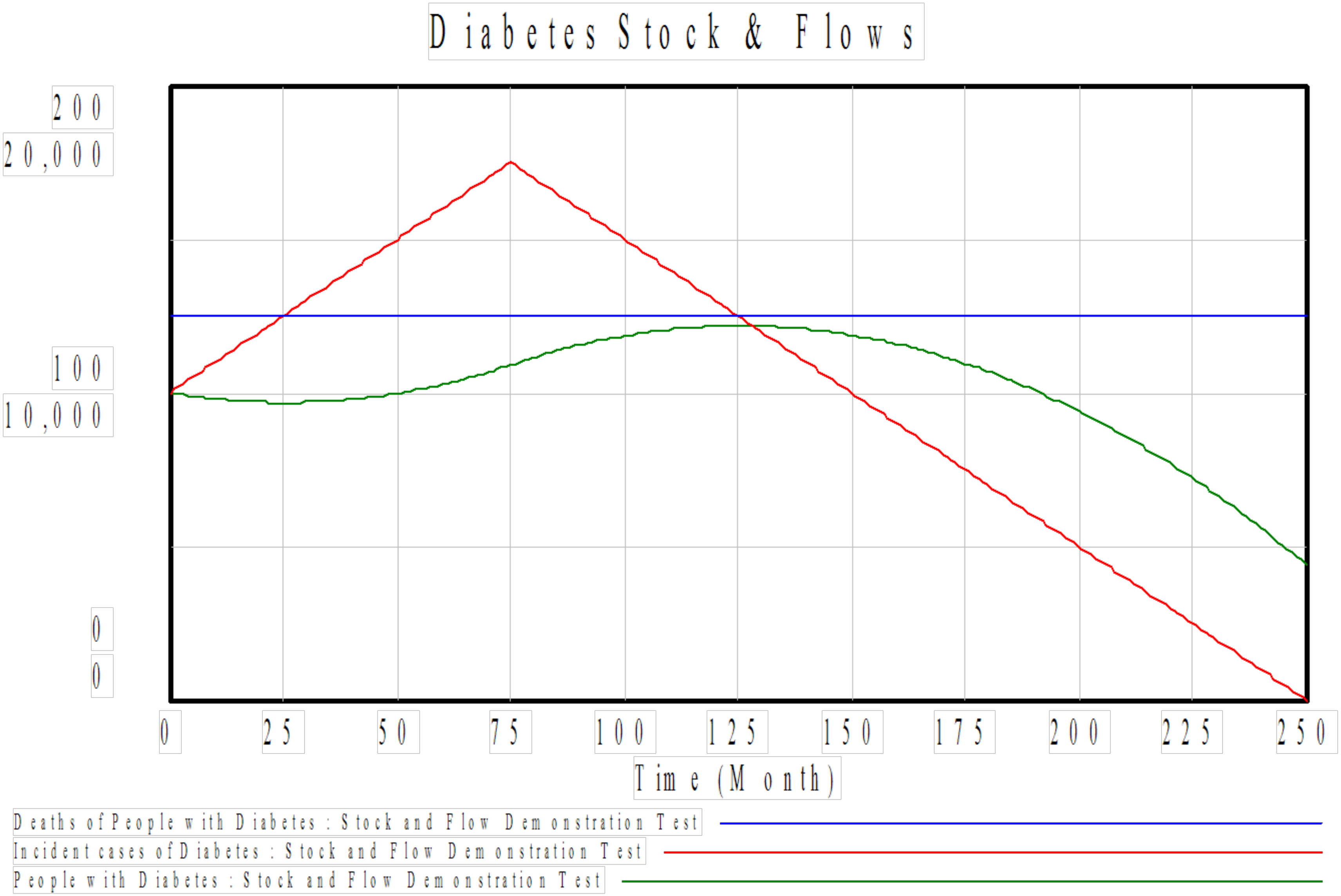
What happens to the stock?

Some Questions



- When is the stock of people with diabetes at its lowest value?
- When is the stock of people with diabetes at its greatest value?
- Is the value of the stock of people with diabetes larger at the beginning or end?
- When is the stock of people with diabetes not changing?

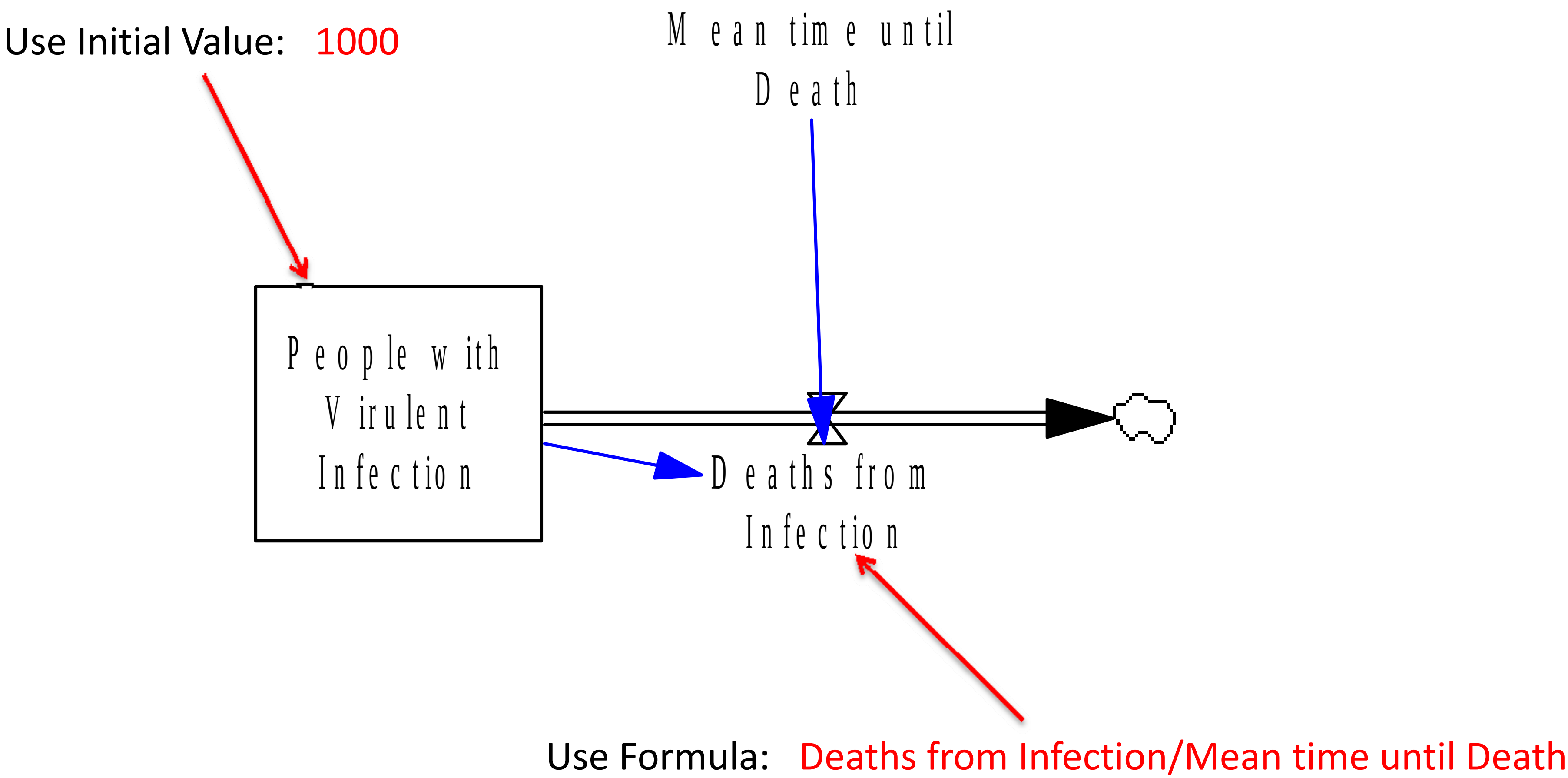
Stocks & Flows



Flows and Feedbacks

- Stocks are always changed by flows
- In your experiments, we've used constant values for flows
- In general, the formulas for the flows will depend on things that are changing (state)
  - Ultimately, these things must depend on the things that keep track of the state – the stocks!

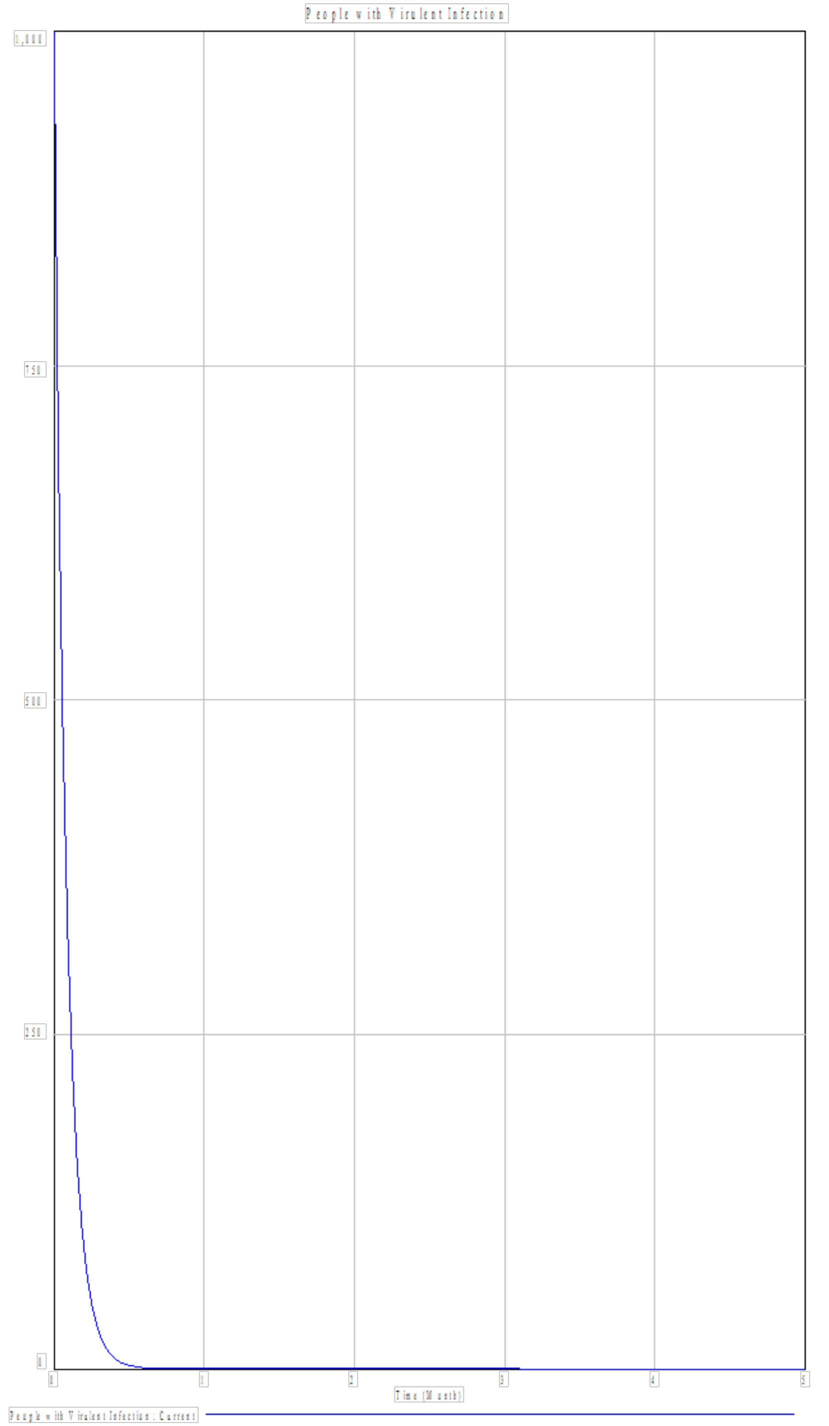
Simple First-Order Decay (Create this in Vensim!)



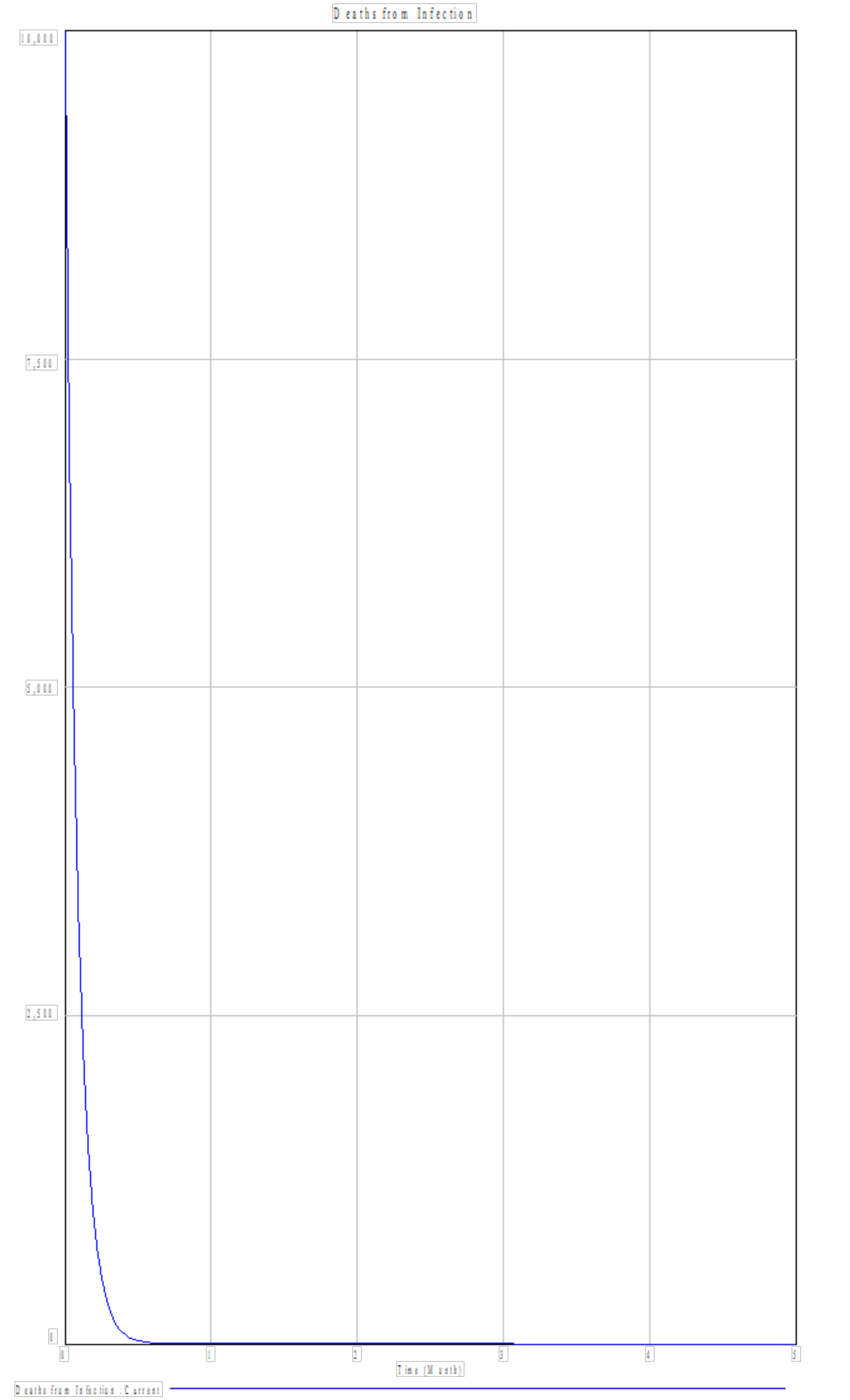
First Order Delays and Transition Processes

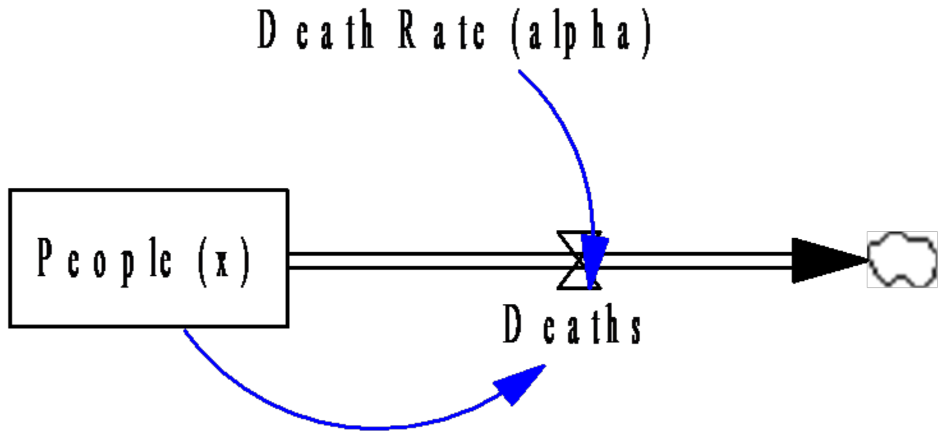
- We can think of first order delays as representing a deterministic approximation to a population experiencing a memoryless (Poisson) stochastic transition process
- The system is "memoryless" because the chance of e.g. a person leaving in the next unit of time is independent of how long they've been there!
- The probability distribution of residence time in the stock is exponentially distributed

Dynamics of Stock?



Dynamics of (Rate of) Death Flow?

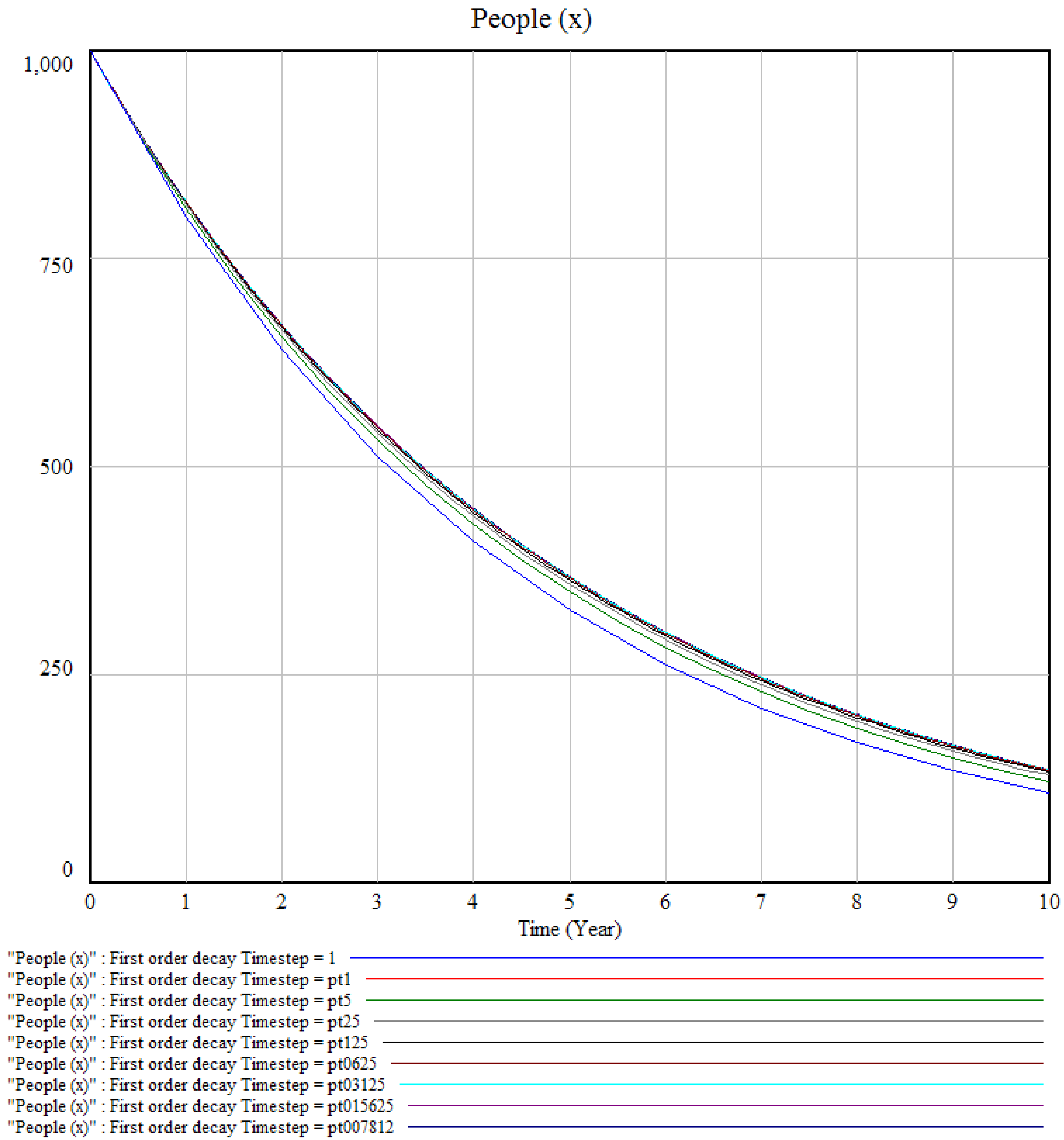




- Alpha is per-time-unit likelihood of death
  - Chance of death over small Δt is $\alpha\Delta t$
  - If x people are at risk, # dying over Δt is x*(Likelihood of death over Δt)=$x(\alpha\Delta t)$= $x\alpha\Delta t$
  - When people die, they flow out => cause a negative change in x.
  - We denote the change in x over the time Δt as Δx

    Thus $\Delta x= -x\alpha\Delta t$
- As x is depleted (becomes smaller), Δx becomes smaller as well (for a fixed Δt)

Impact of Step Size on Simulation



Mean Time

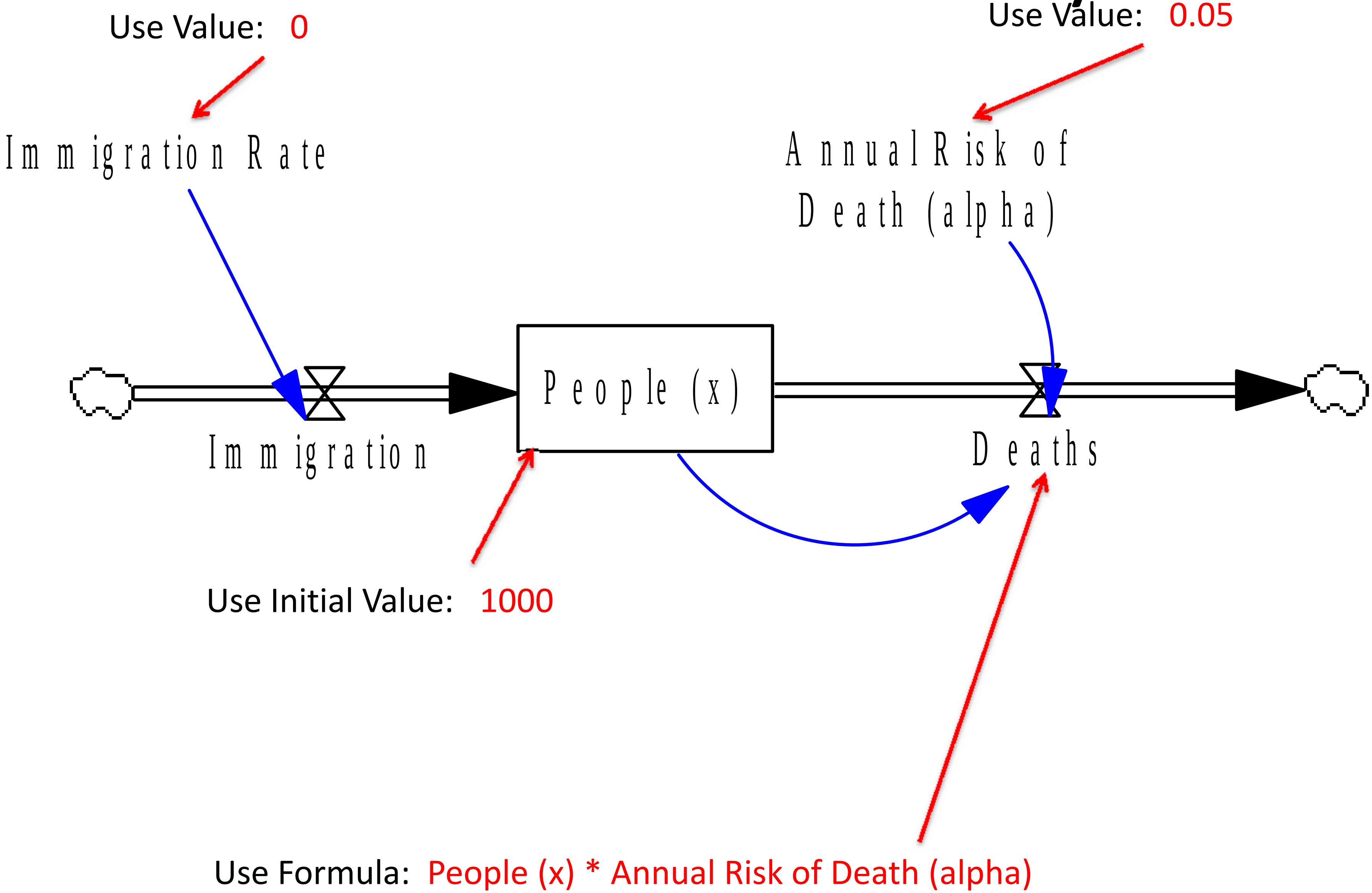
- The mean time (the *delay associated with a first order delay with coefficient α)* is given by

$$E\left[p(t)\right] = \alpha \int_{t=0}^{t=\infty} te^{-\alpha T} dt = \alpha \left(\int_{t=0}^{t=\infty} te^{-\alpha T} dt \right)$$

$$= \alpha \left(\frac{1}{\alpha^2} \right) = \frac{1}{\alpha}$$

- So e.g. if we have an annualized rate of diabetes incidence (e.g. 0.05), the mean time to develop diabetes (independent of other risks) is just the reciprocal of that rate (i.e. 1 over that rate); here, 20 years

Recall: First Order Delay

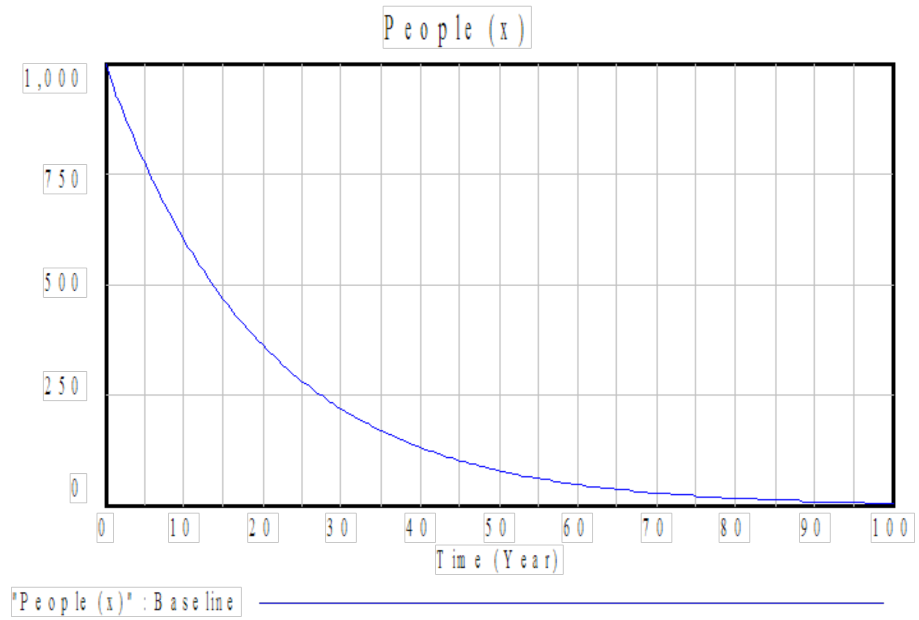


Questions

- What is behaviour of stock x?
- What is the mean time until people die?
- Suppose we had a constant inflow – what is the behaviour then?

Answers

- Behaviour Of Stock



- Mean Time Until Death

  Recall that if coefficient of first order delay is α , then mean time is $1/\alpha$ (Here, 1/0.05 = 20 years)

Equilibrium Value of a First-Order Delay

- Suppose we have flow of rate i into a stock with a first-order delay out
  - This could be from just a single flow, or many flows
- The value of the stock will approach an equilibrium where inflow=outflow

Equilibrium Value of 1st Order Delay

- Recall: Outflow rate for 1st order delay=αx
  - Note that this depends on the value of the stock!
- Inflow rate=i
- At equilibrium, the level of the stock must be such that inflow=outflow
  - For our case, we have

$\alpha x = i$

Thus $x = i/\alpha$

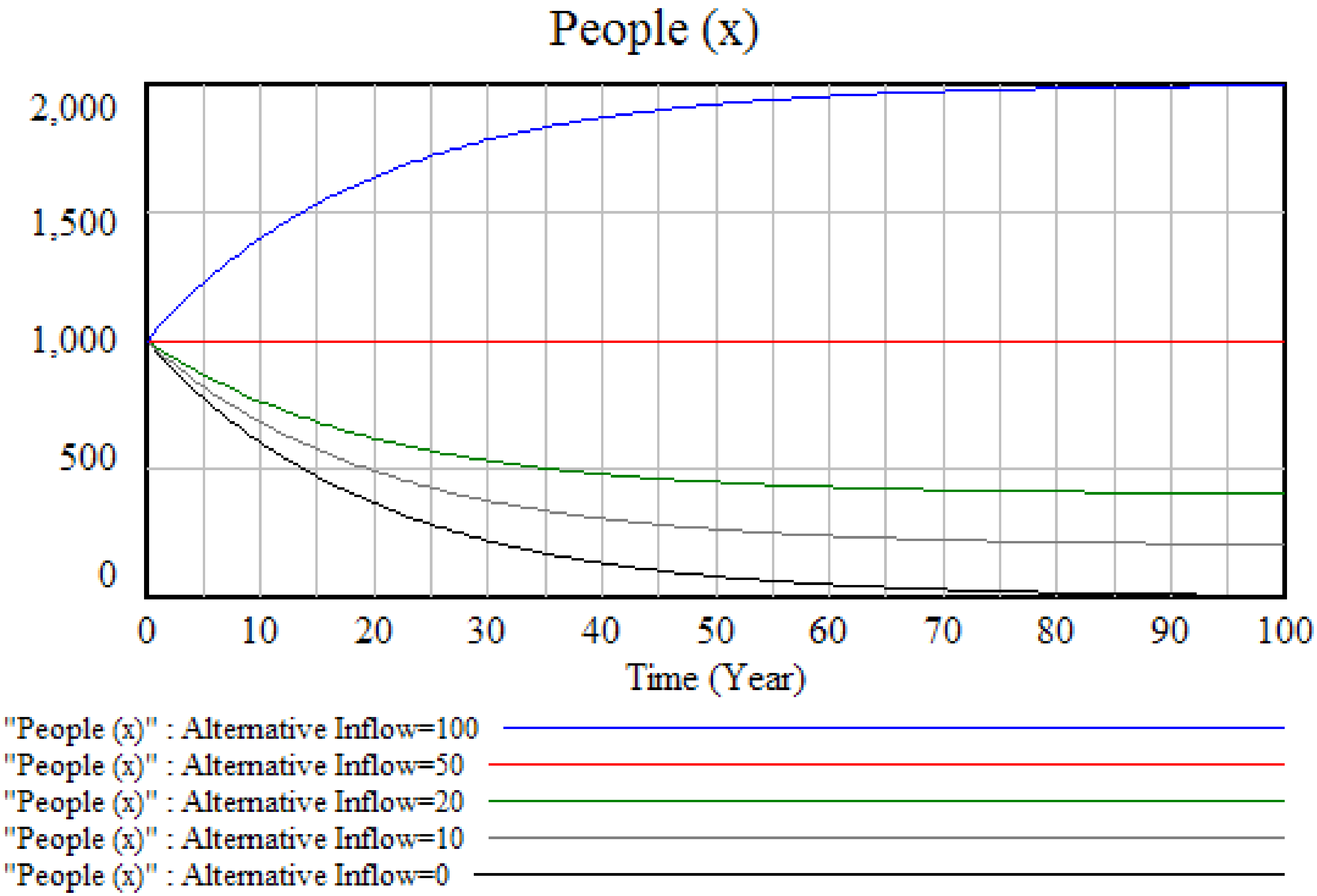
(equivalently, x = i* Mean time to Transition)

The lower the chance of leaving per time unit (or the longer the delay), the larger the equilibrium value of the stock must be to make outflow=inflow

Scenarios for First Order Delay: Variation in Inflow Rates

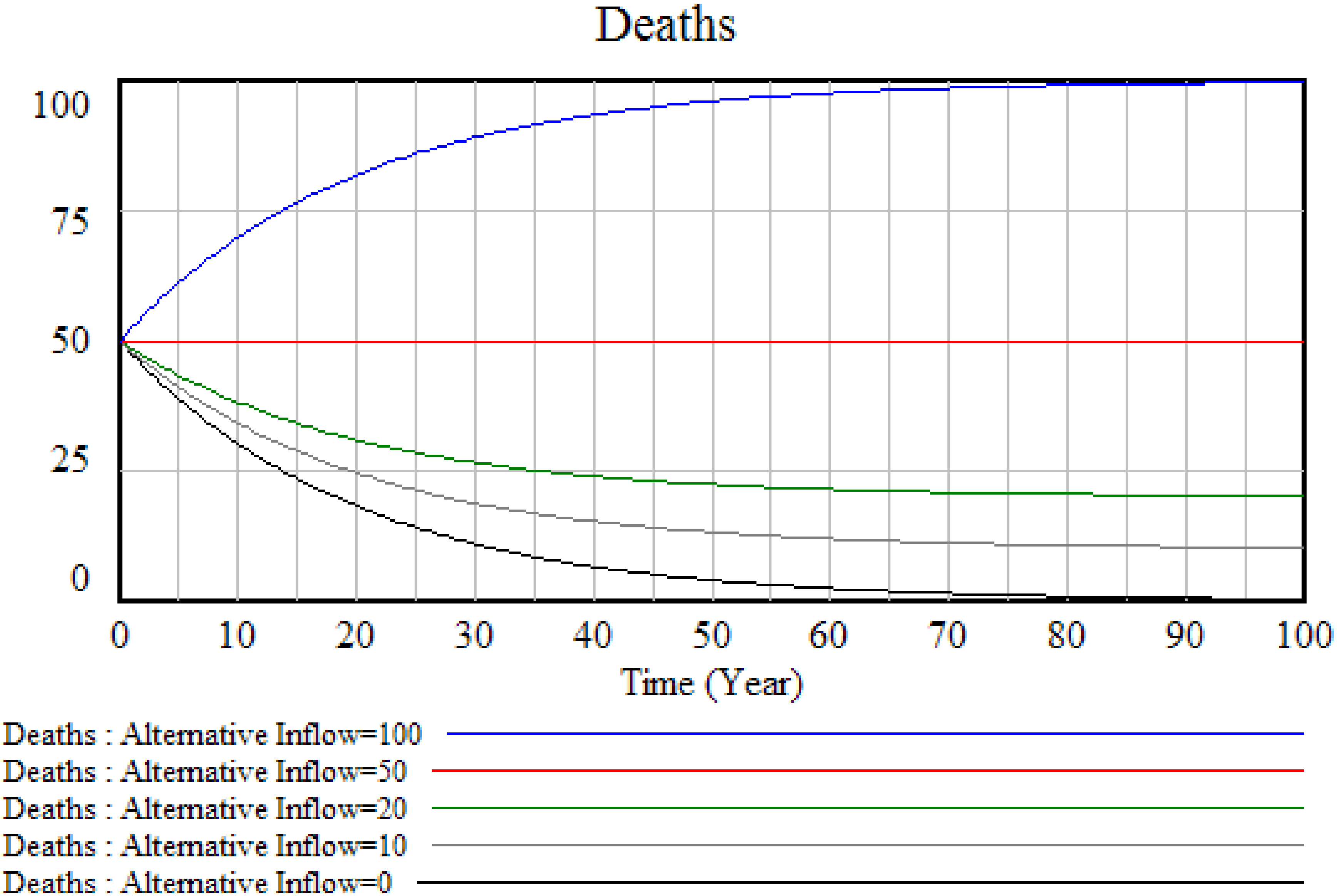
- For different immigration (inflows) (what do you expect?)
  - Inflow=10
  - Inflow=20
  - Inflow=50
  - Inflow=100
  - Why do you see this “goal seeking” pattern?
  - What is the “goal” being sought?

Behaviour of Stock for Different Inflows



Why do we see this behaviour?

Behaviour of *Outflow* for Different Inflows



Why do we see this behaviour? Imbalance (gap) causes change to stock (rise or fall) ⇒ change to outflow to lower gap **until outflow=inflow**

Goal Seeking Behaviour

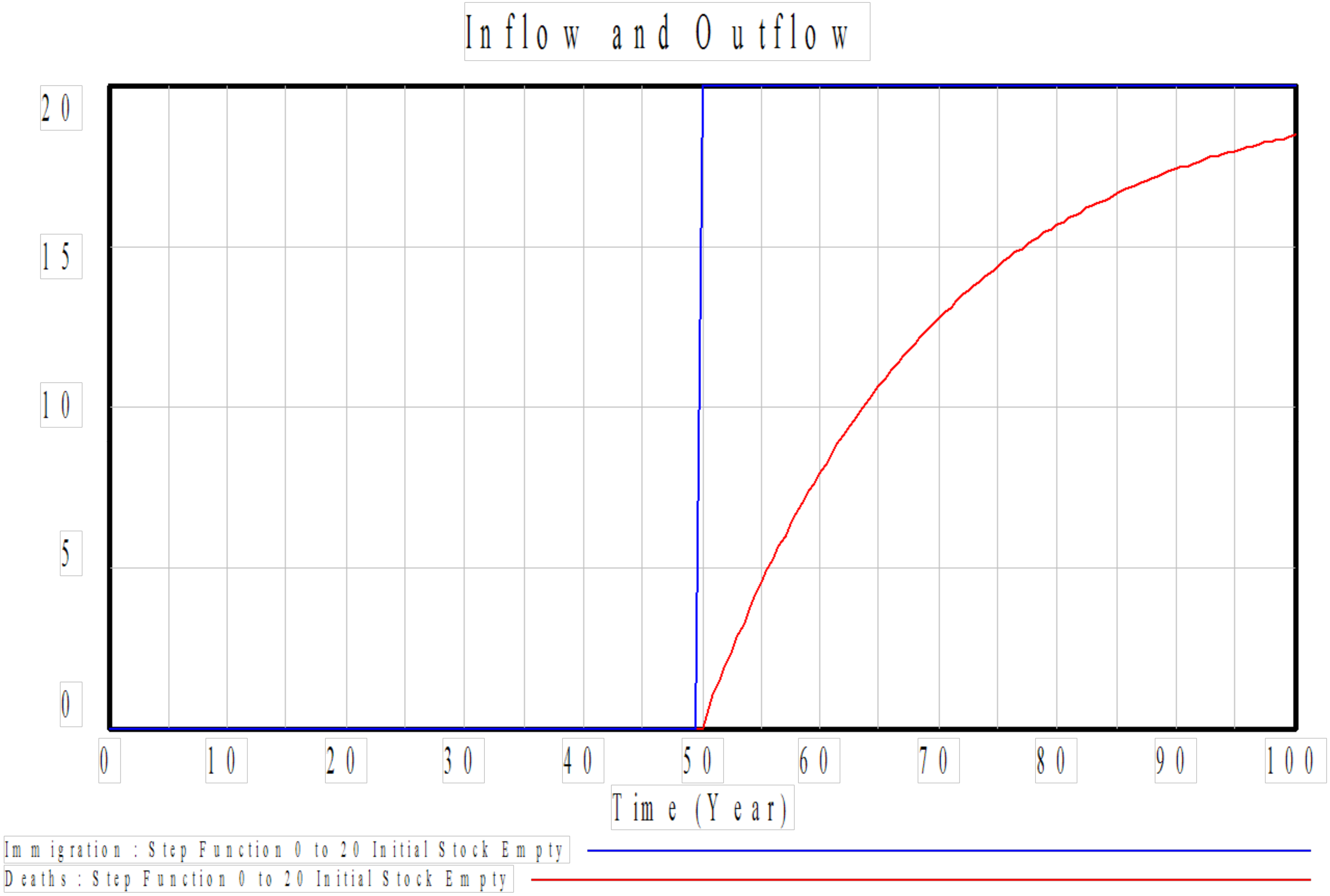
- The goal seeking behaviour is associated with a negative feedback loop
  - The larger the population in the stock, the more people die per year
- If we have more people coming in than are going out per year, the stock (and, hence, outflow!) rises until the point where inflow=outflows
- If we have fewer people coming in than are going out per year, the stock declines (& outflow) declines until the point where inflow=outflows

Response to a Change

- Feed in an immigration “step function” that rises suddenly from 0 to 20 at time 50
- Set the Initial Value of Stock to 0
- How does the stock change over time?

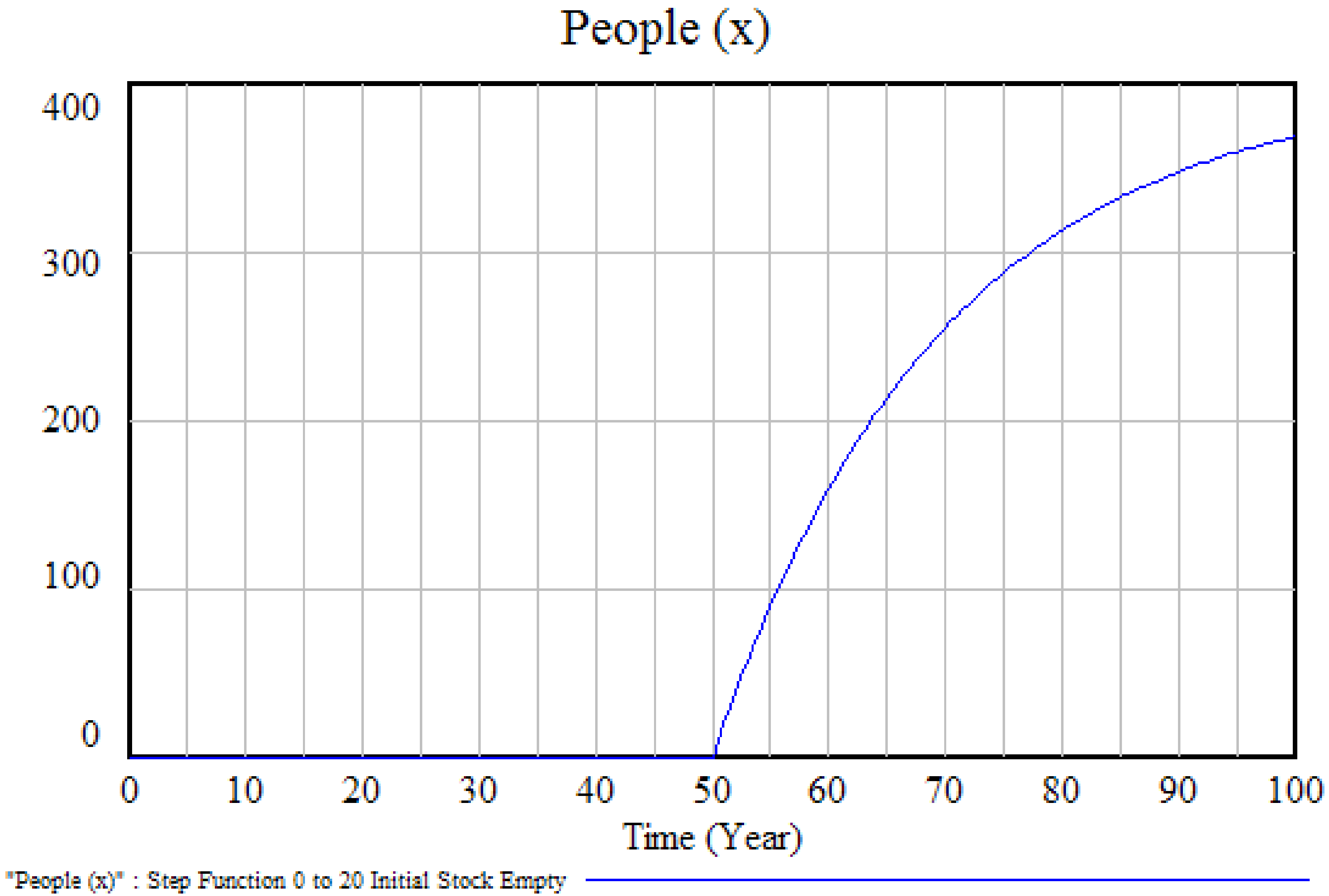
Stock Starting Empty

Flow Rates



How would this change with alpha?

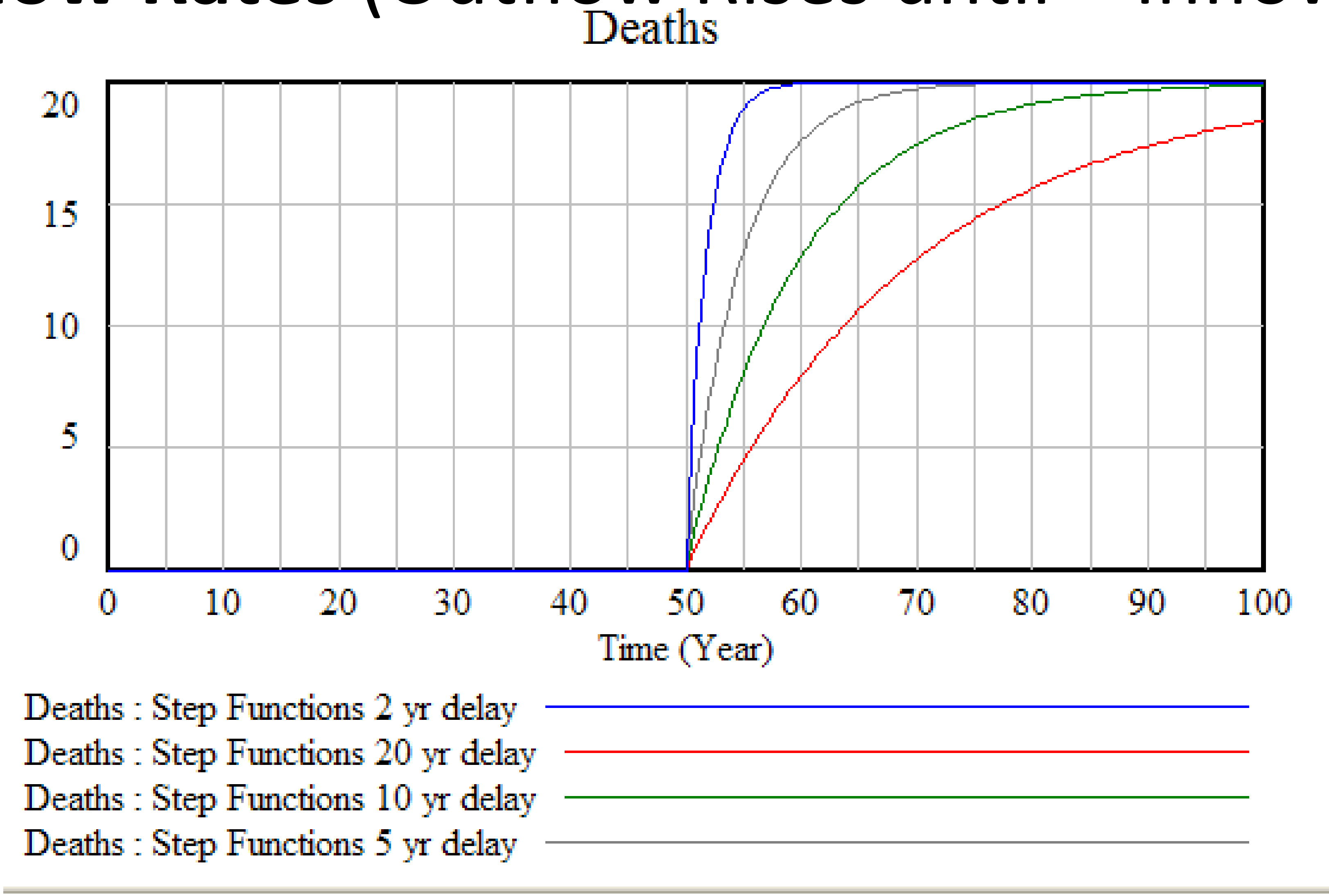
Stock Starting Empty?
Value of *Stock* (Alpha=.05)



How would this change with alpha?

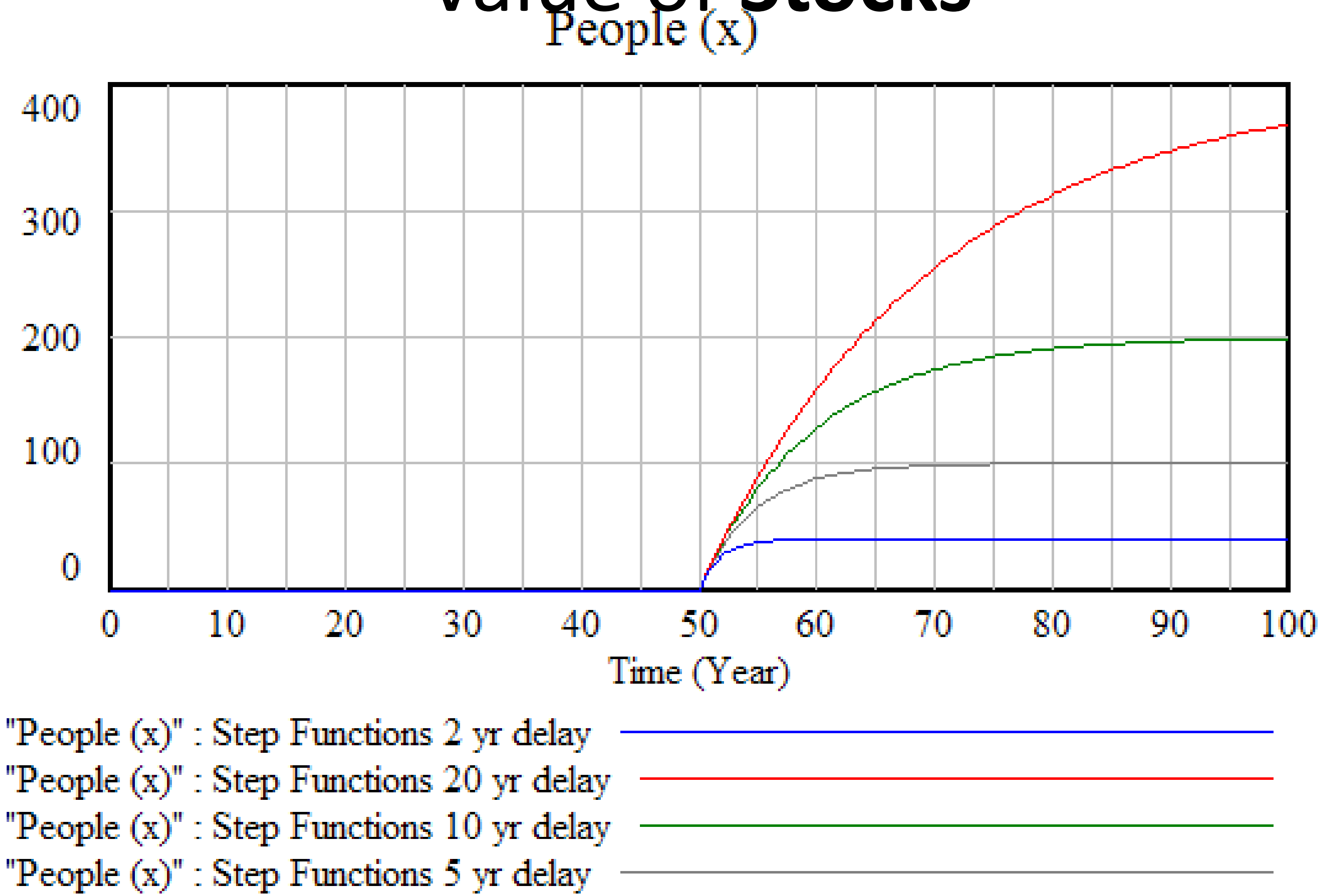
For Different Values of (1/) Alpha

Flow Rates (Outflow Rises until = Inflow)



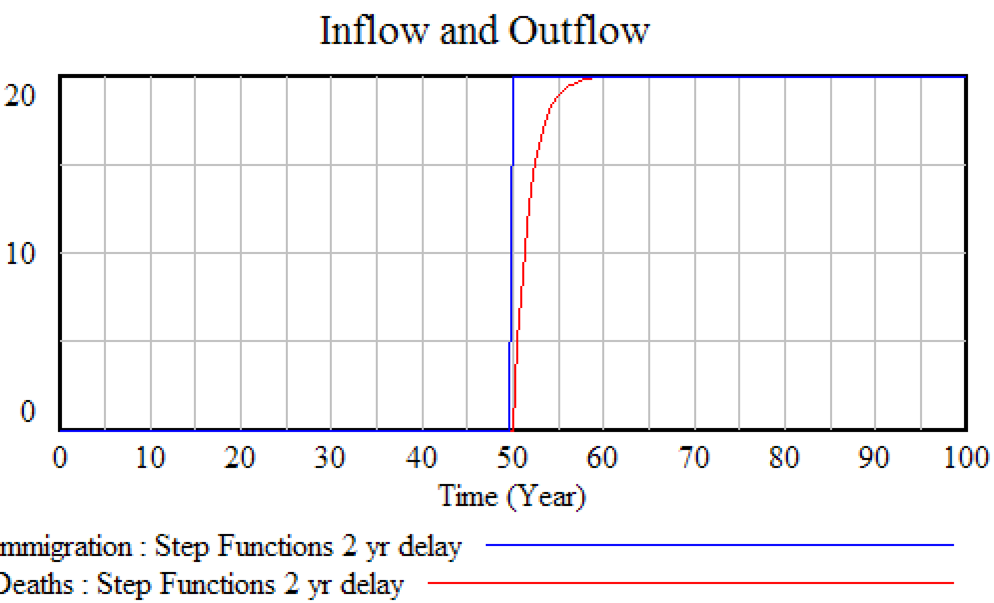
This is for the *flows*. What do stocks do?

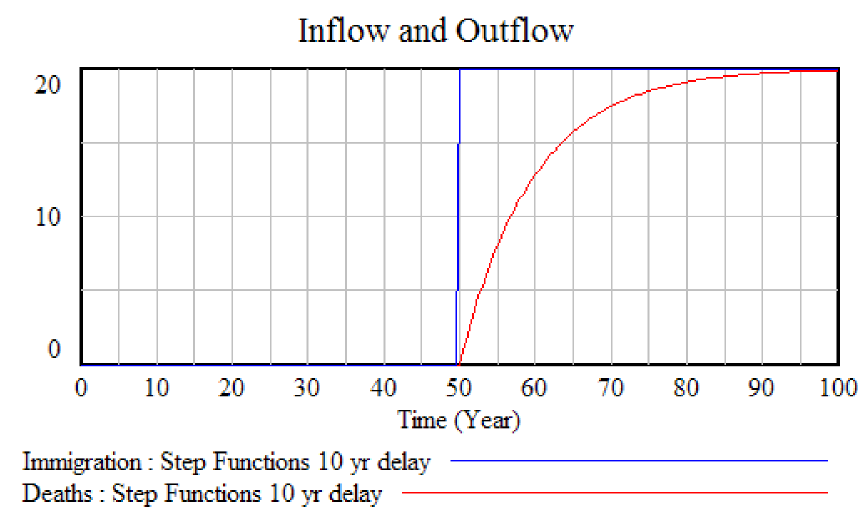
For Different Values of (1/) Alpha
Value of **Stocks**

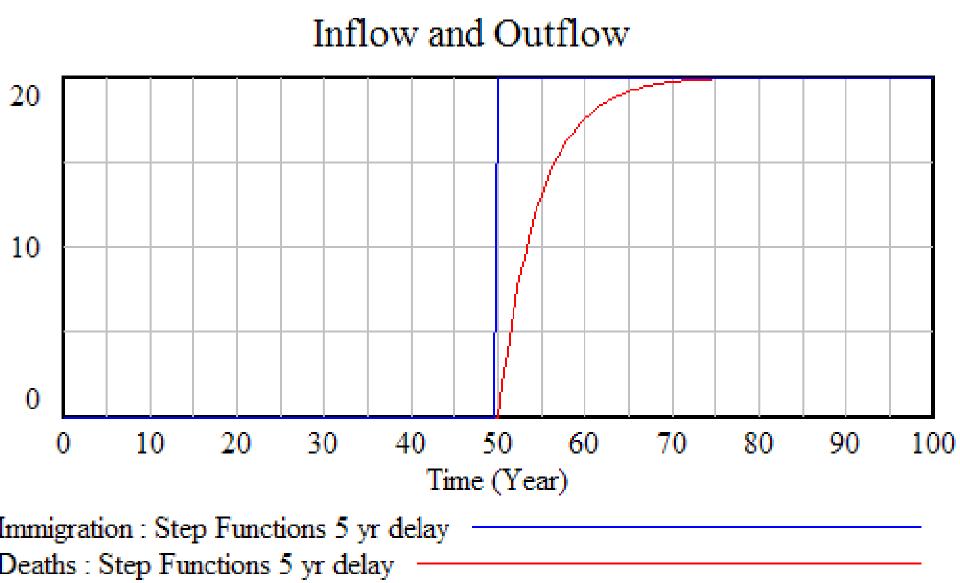


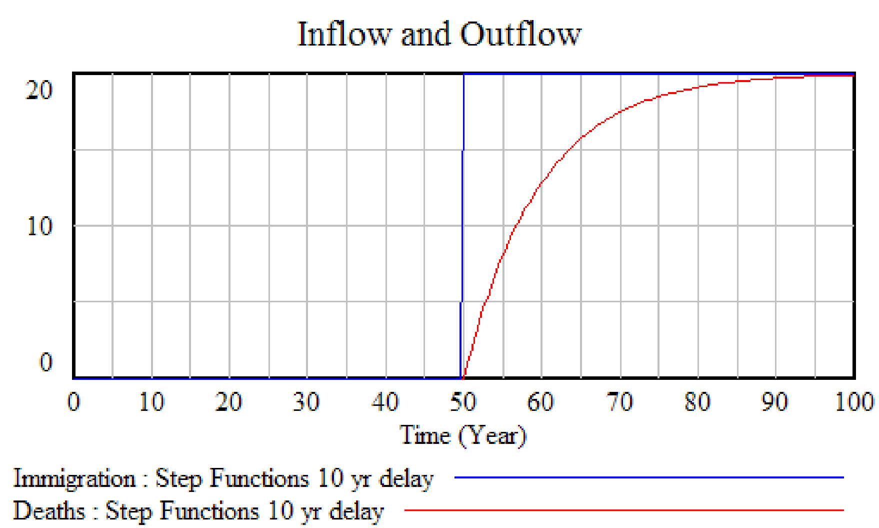
Why do we see this behaviour? A longer time delay (or smaller chance of leaving per unit time) requires *x* to be *larger* to make outflow=inflow

Outflows as Delayed Version of Inputs









What if stock doesn't start empty?

Decays at first (no inflow) & then output responds with delayed version of input